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Axial crushing of circular structures with rectangular multi-cell insert

Stefan Tabacu [1]

Automotive Department, Faculty of Mechanics and Technology, University of Pitesti, Târgu din Vale, 1, 110040 Pitesti, Romania

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ABSTRACT

The axial crushing of circular structures with rectangular insert was investigated. Literature available experimental data were used to validate the numerical models of the structures investigated. Results are compared in order to set up the simulation layout. To increase the efficiency of the simulation process a software module was developed and some explanations related to the generation of the numerical model are provided.

Based on the theoretical models of the symmetric crushing of circular tubes and the model developed to investigate multi-cell square thin-walled structures a formula for the evaluation of the mean crushing force for the proposed structure was derived.

Results obtained from the experiments and different numerical simulations of structures with different number of cells, different thicknesses and main dimensions were evaluated and compared with theoretical results. The theoretical model predicted the mean crushing force with and error of 20% in case of structures with a reduced number of cells (four cells) or with a very thin insert while for structures with a higher number of cells or thicker insert (comparable with the thickness of the tube) the predicted value is within 5% range. Some plots based on the theoretical model are presented and can provide the start point for an optimisation process.

1. Introduction

Thin walled structures are used as energy absorbing components in order to enhance vehicle's crashworthiness. Experimental and numerical studies are performed in order to identify efficient structures. Theoretical models are developed as prediction tool used to set up the initial configurations of the structures that in some cases will be extensively investigated using experimental and numerical methods. The actual progress in this field originates from the work of Alexander [1] who developed the collapse model of thin walled circular tubes and derived the first set of equation to predict the crushing parameters further investigated by Abramowicz and Jones [2] and Wierzbicki et al. [3]. Abramowicz and Jones [4], Wierzbicki and Abramowicz [5] and Hayduk and Wierzbicki [6] focused on the investigation of rectangular thin walled structures. Some of the findings in this field, with respect to their time, were summarised and reported by Alghamdi [7]. Thin walled structures were investigated by Fan et al. [8] as hollow tubes with six (hexagon) to sixteen (star) shaped cross section, Guillow et al. [9] for the collapse mode of aluminium tubes, Gupta and Venkatesh [10] focused on the influence geometrical parameters on the collapse mode, Gupta et al. [11] studied the conical shells, Jusuf et al. [12] studied multi wall rectangular shaped, Najafi and Rais-Rohani [13] studied multi-corner tubes, Nia and Hamedani [14] focused on a set of hollow structures with different cross shapes and Nia and Parsapour [15] on structures with a insert forming a single criss-cross, Qi et al. [16] focused on tapered structures, Tarigopula et al. [17] worked on high strength steel hollow sections, Tran et al. [18] for the multi-cell triangular tubes, Wierzbicki et al. [19] investigated in detail the stress profile in thin walled rectangular structures, Zhang et al. [20–22] focused on the triggering efficiency for collapse modes of thin walled structures, Zhang and Huh [23] investigated column structures and Zhang and Zhang [24] studied axial crushing of multi cell columns. Observations, methods and solutions for the numerical models were adopted and used for the current study in order to improve the efficiency of the presented models. Recently researchers like Li et al. [25] and Zhang et al. [26,27] focused on structures with variable thickness and developed parametric studies that originates from thin walled structures with constant thickness. Using fillers to improve energy absorption represents a different solution that was investigated by Ahmad et al. [28], Darvizh et al. [29], Gameiro and Cirne [30], Hanssen et al. [31], Santosa et al. [32], Yin et al. [33] and Zheng et al. [34]. However, multi-wall structures can be defined in order to have a similar output like the functionally graded thickness or foam-filled structures and similarities can be identified [21].

A set of circular tubes with different insert configurations were

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E-mail address: stefan.tabacu@upit.ro

[1] http://www.upit.ro

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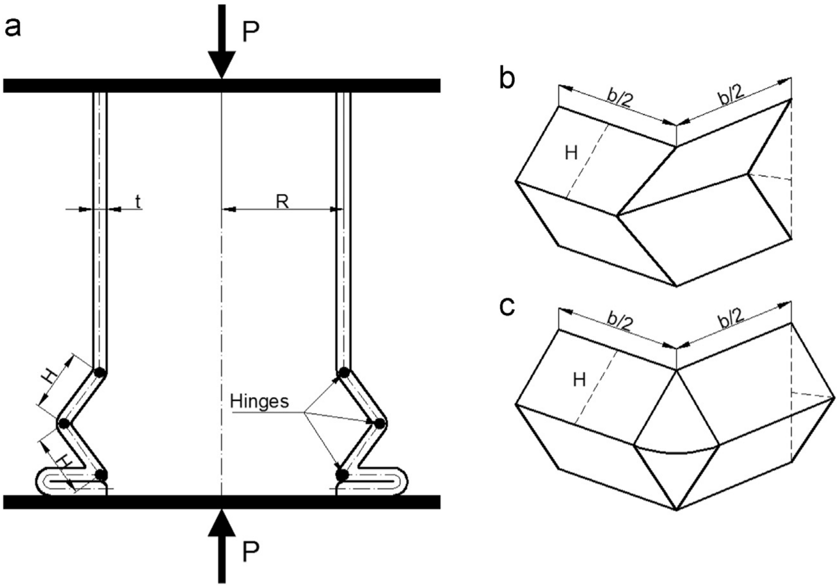

Fig. 1. Axial crushing of tubes. (a) Idealised axisymmetric crushing model of circular tube [2,3,35]; (b) symmetric crushing mode of a rectangular tube [4,5]; (c) extensional crushing model of a rectangular tube.

investigated by Zhang and Zhang [24]. The experimental results provide the background for the validation of the numerical model in this study. For the multi-cell square thin walled structures Zhang et al. [21] derived an equation that is capable to predict with accuracy the mean crushing force. The model uses a simplified model of the Super Folding Element used to investigate de deformation of the corner structures. In order to best fit experimental and numerical results the enhancing coefficient is used in their formula.

In this paper circular structures with rectangular insert are investigated. Based on the above mentioned works a formula capable to predict the mean crushing force is developed and the results are compared with experimental data (when available) or with simulation results. The formula is derived as a combination of the folding model for the symmetric crushing model of circular tubes and the simplified model of the Super Folding Element and following the method proposed by Zhang et al. [21] an enhancing coefficient is used to best fit experimental and simulation results. The value of the enhancing coefficient was determined using the updated model for the evaluation of the mean crushing force in case empty tubes proposed by Wierzbicki et al. [3] and the detailed analysis of the structure as presented by Zhang and Zhang [24]. The formula proposed in this paper is useful for quick estimates of the mean crushing force and some parametric analyses and once a solution of the problem is identified an in-depth analysis and accurate evaluation of the mean crushing force using analytical tools can be obtained following the procedure of Zhang and Zhang [24].

2. Theoretical model of the mean crushing force

2.1. Introductory equations

The mean crushing force for the circular tube, crushing in concertina mode, was defined by a theoretical formula by Alexander [1] as

$$P_m = 6.08 \cdot \sigma_0 \cdot D^{0.5} \cdot t^{0.5} \tag{1}$$

Subsequently and updated was provided by Abramowicz and Jones [4]

$$P_m = \frac{20.79 \cdot (D/t)^{0.5} + 11.9}{0.86 - 0.568 \cdot (t/D)^{0.5}} \cdot M_0 \tag{2}$$

Based on the work of Alexander [1] Wierzbicki et al. [3] proposed the following solution for the mean crushing force:

$$\frac{P_m}{M_0} = 31.74 \cdot \sqrt{\frac{D}{t}} \tag{3}$$

Additional formulae were provided by Guillow et al. [9] and Hanssen et al. [31]

$$P_m = 72.\ 3 \cdot \left(\frac{D}{t}\right)^{0.32} \cdot M_0 \tag{4}$$

and

$$P_m = 17.0 \cdot \sigma_0 \cdot (D - t)^{1/3} \cdot t^{5/3} \tag{5}$$

For the above mentioned equations D denotes the mean diameter of the tube t the wall thickness; σ_0 is the flow stress of the structural material and $M_0 = 1/4 \cdot \sigma_0 \cdot t^2$ is the fully plastic bending moment per unit width.

The flow stress can be calculated by using the strain hardening exponent n from the power law stress strain curve as follows [16,21,24,31]:

$$\sigma_0 = \sqrt{\frac{\sigma_y \cdot \sigma_u}{1 + n}} \tag{6}$$

For the rectangular tubes Abramowicz and Jones [4] derived the expression of the mean crushing force (P_m) as:

$$P_m = 13.06 \cdot \sigma_0 \cdot b^{1/3} \cdot t^{5/3} \tag{7}$$

where b and t are the width and wall thickness of tube. The mean crushing force was evaluated by Wierzbicki et al. [3] as

$$P_m = 3 \cdot \pi \cdot M_0 \cdot \left(\frac{b}{t}\right)^{1/3} \tag{8}$$

Fig. 1 presents the crushing model for circular and rectangular tubes used to derive the expressions of the mean crushing force.

Based on the Super Folding Element [5], Zhang et al. [21] derived, for multi-cell rectangular tubes, the theoretical prediction of