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An investigation into the plastic buckling paradox for circular cylindrical shells under non-proportional loading

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ABSTRACT

Many authors in the literature agreed that the flow theory of plasticity either fails to predict buckling or overestimates plastic buckling stresses and strains of plates and shells while the deformation theory succeeds in forecasting buckling and provides estimates that are more in line with the experimental results. Following a previous study by the same authors focused on compressed cylinders, the present work aims to investigate the reasons for the discrepancy between the flow and deformation theory predictions in the case of cylinders subjected to combined axial tensile load and increasing external lateral pressure. To this end, geometrically nonlinear finite-element calculations of selected cylindrical shells using both the flow theory and the deformation theory of plasticity have been conducted, and the results are compared with some accurate physical test results and with numerical results obtained by other authors using the code BOSOR5. It is found, contrary to common belief, that very good agreement between numerical and test results can be obtained in the case of the flow theory of plasticity. The reasons underlying the apparent plastic buckling paradox are discussed in detail.

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1. Introduction

Plastic buckling of circular cylindrical shells has been the subject of intense research for many decades due to its importance in many engineering applications. In particular, many experimental, analytical and numerical studies have been conducted to investigate the buckling of cylinders subject to axial compression, external pressure, torsion or combination of such load cases.

In general, accurate numerical or analytical estimation of the critical load of plastic buckling of real cylinders requires accounting for moderate large deflection and, nonlinear, irreversible (i.e. path-dependent) material behaviour [6].

As for the material constitutive law to be used, the plasticity models that have been proposed for metals fall within one of two different theories: the ‘deformation theory’ of plasticity and the ‘flow theory’ of plasticity. In both of these theories the plastic-strain increments are isochoric, i.e. characterised by zero volume change, and their evolution is governed by the second invariant J_2 of the deviatoric part of the stress. The flow theory of plasticity defines a path-dependent law, in which the current stress depends not only on the value of the current total strain but also on how the actual strain value has been reached. On the other hand, the deformation theory of plasticity is based on the assumption that for continued loading the state of stress is uniquely determined by the current state of strain and, therefore, it is a special class of path-independent non-linear elastic constitutive laws.

Experimental investigations show that plastic strains depend both on the the stress value and the loading history. Thus, there is general agreement among engineers and researchers that the deformation theory of plasticity lacks of physical rigour with respect to the flow theory. However many authors, such as Onat and Drucker [17], Mao and Lu [16], Ore and Durban [18] and Bardi and Kyriakides [2], among the others, pointed out that the deformation theory tends to predict buckling loads that are smaller than those obtained by the flow theory and much closer to the experimental results. In fact, the flow theory seems to over-estimate buckling loads, often quite significantly.

There have been many attempts to explain this so called "plastic buckling paradox" and formulate accurate methods based on the flow theory of plasticity, that typically differ from each other on account of the choice and formulation of the constitutive equations and of the associated factors. For instance, Batdorf and Budiansky [3] suggested to use the slip theory in plastic buckling analysis. Sewell [23] proposed the use of Tresca yield surface in the flow theory of plasticity and Lay [15] proposed that the effective shear modulus should be employed when using the flow theory, whereas Ambartsumjan [1] recommended considering the transverse shear deformation. Attention was also paid to the consideration of initial imperfections, as proposed by Onat and Drucker [17].

More recently, Shamass et al. [21] numerically investigated buckling of axially compressed cylindrical shells in the plastic range. They showed that non-linear finite-element buckling analyses based on the flow theory provide buckling stresses in better agreement with the experimental results than those based on the deformation theory, a fact that is in contrast with the conclusions by Mao and Lu [16], Ore and Durban [18] and Bardi and Kyriakides [2]. Shamass et al. [21] concluded that the main root of the discrepancy between the two plasticity theories can be found in the assumptions made in many analytical treatments with regard to the shape of the buckling modes, a simplification which gives origin to an excessively constrained kinematics, in turn counterbalanced by the material description of the deformation theory of plasticity. This fact has also been confirmed to a certain extent by analytical investigations [22].

In the case of axially loaded cylinders, at least during the elastic phase, the walls are subjected to proportional loading, and in many points during plastic yielding the deviation from the loading path is relatively limited. Nevertheless, the flow and deformation theory seem to provide quite different results.

It is therefore not surprising that similar or even more significant discrepancies have been reported between the results from the flow and deformation theory in the case of non-proportional loading even in the elastic phase.

Blachut et al. [4] conducted experimental and numerical analyses on 30 mild-steel machined cylinders of different dimensions, subject to axial tension and increasing external pressure. Using the code BOSOR5 [6] for their numerical analyses they showed that the agreement between the two plasticity theories was strongly dependent on the length of the cylindrical shell. For short cylinders ($D/L = 1$), the plastic buckling results predicted by the flow and deformation theory coincided only when the tensile axial load vanished. By increasing the axial tensile load, the buckling pressures predicted by the flow theory started to diverge quickly from those predicted by the deformation theory. Additionally, the flow theory failed to predict buckling for high axial tensile load while tests confirmed the buckling occurrence. For specimens with length-to-diameter ratio L/D ranging from 1.5 to 2.0 the results predicted by both theories were identical for a certain range of combined loading. However, for high values of the applied tensile load, the predictions of the flow theory began to deviate from those of the deformation theory and became unrealistic in correspondence to large plastic strains.

Giezen et al. [12] conducted experiments and numerical analyses on two sets of tubes made of aluminium alloy 6061-T4 and subjected to combined axial tension and external pressure, once again making resort to the code BOSOR5 [6]. The tubes were characterised by a L/D ratio equal to one and two loading paths were considered. In the first one the axial tensile load was held constant and the external pressure was increased. In the second one, the external pressure was held constant and the axial tensile load was increased. The numerical studies showed that the buckling pressure predicted by the flow theory increases with increasing applied tensile load while the experimental tests showed on the contrary a reduction in buckling resistance with increasing axial tension. Thus, the discrepancy between the test results and the numerical results predicted by the flow theory increased significantly with the intensification of the axial tension. On the other hand, the results by the deformation theory displayed the same trend of the test results. However, the deformation theory significantly under-predicted the buckling pressure observed experimentally for some load-paths. Therefore, Giezen [13] concluded that both plasticity theories were unsuccessful in predicting buckling load. Interestingly enough, Giezen showed in his thesis [13] that, when reversing the load path, the deformation theory was able to predict buckling while the flow theory failed to do so.