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Influence of bimoment on the torsional and flexural–torsional elastic stability of thin-walled beams

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ABSTRACT

This article studies the influence of the bimoment on the buckling of thin-walled beams with open cross section subjected to axial loading. In the case of torsional and torsional-flexural buckling of thin-walled Z-section beam, it is shown that influence of the bimoment could be of importance in the assessment of buckling loads. In order to verify the accuracy and validity of this analysis, the obtained results are compared with those calculated by ANSYS software.

1. Introduction

Thin-walled beams are widely used as structural elements in many types of structures. This is due to the fact that structural members with such a cross section may have a high load-carrying capacity, as compared to their self-weight, combined with adequate stiffness. Elastic stability (buckling) of such beams is one of the most important criteria in the design of any structure. This topic is an area of extensive research and is covered in various textbooks, e.g. [1–6], including classical book by Vlasov [7]. Recently, a lot of effort has been done in order to investigate the phenomena that the classical thin-walled beam theories (mainly based on Vlasov's assumptions) are unable to consider, such as the distortion of the cross section including local and distortional buckling. The majority of this research is based on the finite element and the finite strip method, e.g. Ádány and Schafer [8] or on the generalized beam theory – GBT, e.g. [9–10].

This article studies a global buckling of a beam considering the assumptions of the classical thin-walled beam theory with an open cross section. Our attention is focused on the influence of the bimoment induced by axial loads as a part of a fundamental stress field on the bifurcation stability. The existence of the prebuckling bimoment in the equations of torsional stability is recognized by several authors, e.g. [7] and [11–13], but without numerical evidence of the importance of this term. Yoo [14] included the bimoment in the one-dimensional finite element model for the analysis of eccentrically loaded thin-walled beam. Kim et al. [15] demonstrated, also by the one-dimensional FE model, the influence of bimoment in the case of lateral buckling of cantilever (with asymmetric cross section) due to eccentric lateral load. The lack of interest for this phenomenon in the compressed thin-walled beams could be partially explained by the fact that for all cross sections with at least one axis of symmetry the influence of bimoment on the stability does not exist. In this article the influence of the prebuckling bimoment on the torsional and the flexural–torsional stability (i.e. buckling with coupled bending and torsion in the presence of eccentric axial force) is investigated through the analysis of a thin-walled Z-section beam.

2. Formulation and application

Consider a straight thin-walled beam with uniform open cross section. Besides the usual assumptions of the linear theory of elasticity, the classical Vlasov's assumptions of the cross section kinematics are used:

(a) Cross section is assumed to be perfectly rigid in its own plane;
(b) Shear deformation in the middle surface of each thin-walled plate is neglected;
(c) There is no shear strain in the plane perpendicular to the middle surface (Kirchhoff 's thin plate bending assumption).

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The assumption (a) is ensured by the plate thickness-width ratio or by transverse diaphragms and stiffeners (e.g. in bridge engineering). If this is not a case, the distortion and/or local buckling can modify the global buckling force. An insight into the influence of assumptions (b) and (c) in the buckling analysis could be found in the recent numerical studies by Ádány and Visy [16].

The governing equilibrium equations of a thin-walled beam according to the linearized second-order theory and corresponding notations are given in the Appendix A, while the geometric parameters, sectional forces and external loads are specified in the Appendix B. For a simply supported beam subjected to an eccentric compressive force P at both ends, the following pre-buckling internal forces are produced: a constant axial force ${}^0N=-P$, bending moments ${}^0M_x=-Pe_x$, ${}^0M_y=-Pe_y$ (e_x, e_y are eccentricities of the loading point) and a variable bimoment ${}^0M_\omega(z)=\lambda(z){}^0M_{\omega 0}$ (${}^0M_{\omega 0}=-P\,\omega_{(P)}$ denotes the bimoment at the beam ends due to axial force P, $\omega_{(P)}$ is the value of warping function at the loading point, while $\lambda(z)$ is the bimoment distribution function). The buckling load is defined by the coupled homogeneous eqs. (A11)$_{2-4}$, from the Appendix A which in the studied case could be expressed in the form

$$\begin{aligned}
&EI_{xx}u_D'''' + Pu_D'' + P(y_D-e_y)\varphi''=0\\
&EI_{yy}v_D'''' + Pv_D'' - P(x_D-e_x)\varphi''=0\\
&EI_{\omega\omega}\varphi'''' - GK\varphi'' + P\left(i_D^2+2e_x\beta_x+2e_y\beta_y\right)\varphi'' + P\omega_{(P)}\beta_\omega(\lambda\varphi')'\\
&\qquad + P(y_D-e_y)u_D'' - P(x_D-e_x)v_D''=0
\end{aligned} \tag{1}$$

where $u_D(z)$, $v_D(z)$ and $\varphi(z)$ denote lateral buckling displacements of the shear center D and the buckling rotation of the cross section around D. In Eq. (1), the deformations prior to buckling are assumed to be sufficiently small and are neglected.

The bimoment distribution function $\lambda(z)$ is determined in the pre-buckled configuration by the solution of the differential equation of warping torsion according the first-order theory, Murray [3], due to unit bimoments at the ends of a thin-walled beam

$$\lambda(z)=\cosh(kz)+\frac{1-\cosh(kl)}{\sinh(kl)}\sinh(kz)\quad k=\sqrt{GK/EI_{\omega\omega}} \tag{2}$$

where l denotes the beam length. In this article, in order to obtain a closed form solution and an engineering assessment of buckling

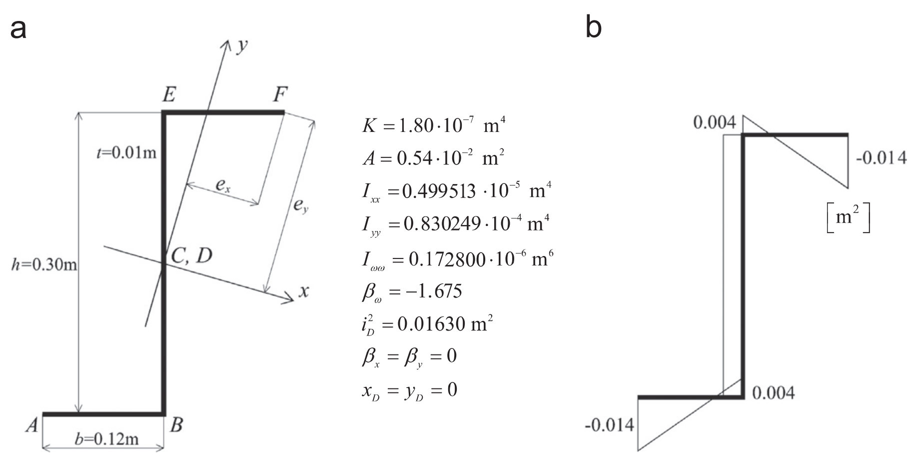
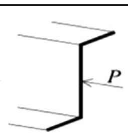
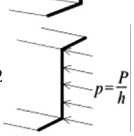
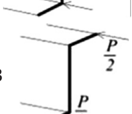
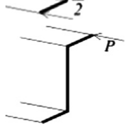


Fig. 1. Thin walled Z-section: (a) Cross section layout with geometric properties for $h=0{,}30$ m, $b=0{,}12$ m $t=0{,}01$ m, (b) diagram of warping function.

Table 1
Buckling loads P_{cr} [MN]: torsional (cases 1 to 3) and flexural-torsional (case 4) $h=0{,}30$ m, $b=0{,}12$ m, $t=0{,}01$ m.

CASE	$l=4.0$ m			$l=6.0$ m			$l=8.0$ m		
	Ansys	Theory $\lambda=\lambda_m$ $\lambda=0$	$\frac{\text{Theory}}{\text{Ansys}}$	Ansys	Theory $\lambda=\lambda_m$ $\lambda=0$	$\frac{\text{Theory}}{\text{Ansys}}$	Ansys	Theory $\lambda=\lambda_m$ $\lambda=0$	$\frac{\text{Theory}}{\text{Ansys}}$
1 	3.5366	3.1320 2.2652	0.886 0.641	2.1214	1.8939 1.5023	0.893 0.708	1.6273	1.4711 1.2352	0.904 0.759
2 	3.4203	3.1320 2.2652	0.916 0.662	2.0733	1.8939 1.5023	0.913 0.724	1.6010	1.4711 1.2352	0.919 0.771
3 	1.0513	1.1504 2.2652	1.094 2.155	0.7661	0.8712 1.5023	1.137 1.961	0.6718	0.7912 1.2352	1.178 1.839
4 	0.3987	0.4162 0.4414	1.044 1.107	0.2039	0.2119 0.2172	1.039 1.065	0.1257	0.1304 0.1318	1.037 1.049