Thin-Walled Structures 88 (2015) 58–69

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Ultimate behaviour of continuous steel beams with discrete lateral restraints

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ARTICLE INFO

Article history:
Received 5 November 2014
Accepted 10 November 2014
Available online 18 December 2014

Keywords:
Continuous beams
Strain-hardening
Lateral restraint forces

ABSTRACT

Through a programme of experiments, numerical modelling and parametric studies, the implications of allowing for strain-hardening in the design of laterally restrained continuous steel beams are investigated with particular emphasis on the performance of the bracing elements. A total of six tests were performed on continuous beams considering two basic scenarios: discrete rigid restraints and discrete elastic restraints of varying stiffness. In the latter case, the forces developed in the restraints were measured and compared to the design forces specified in EN 1993-1-1 (2005) for members containing rotated plastic hinges. Two different restraint spacings were considered in the tests to give non-dimensional lateral torsional slenderness values of 0.3 and 0.4 for the unrestrained lengths. In all tests, bending resistances predicted by the deformation-based continuous strength method (CSM) were exceeded. Using a standardised numerical model validated against the laboratory test data, a series of parametric studies were conducted; it was concluded that elastic restraints for members containing rotated plastic hinges should be designed to sustain higher forces than required for traditional plastic design if the full CSM collapse load is to be achieved.

1. Introduction

The resistance of statically indeterminate beams to lateral instability can be improved through the provision of effective lateral bracing, either continuously or at intervals along the length of the member. For discrete bracing systems, the spacing of the lateral restraints influences the bending resistance of the member. In order to be effective, the restraints should have adequate stiffness to limit the lateral displacements at the point of restraint and have sufficient strength to withstand the forces that arise as a consequence of these displacements as well as any initial imperfections. It was shown by Winter [1] that, provided the restraint is of adequate stiffness, these forces are small relative to the axial forces in the primary member.

In the traditional plastic design of statically indeterminate structures, the final collapse mechanism develops through the sequential formation of plastic hinges. In order for subsequent hinges to form, the preceding plastic hinges are required to rotate. Once the hinge has formed, there is a reduction in stiffness and no further spread of yield. At these rotating plastic hinges, additional demands will be placed upon the restraints compared with statically determinate structures, which will not contain rotating plastic hinges. Provisions in EN 1993-1-1 reflect this increased demand and minimum restraint force resistances have been stipulated for the case of traditional plastic design. However, this provision is yet to be verified for a newly proposed, deformation-based design procedure, referred to herein as the continuous strength method (CSM), where moments beyond the full plastic capacity can be achieved through allowing for strain-hardening [2].

Numerous studies of lateral restraint requirements have been carried out [3–14], typically considering elastic member behaviour. The present research is devoted to examining the lateral stability implications of allowing for plasticity and strain-hardening in the design of the primary members. To this end, a series of experiments on continuous beams with variations in restraint spacing and stiffness were conducted. Using a geometrically and materially non-linear finite element model, the test data were reproduced and extended in a parametric study which was then used to inform and develop some basic design recommendations.

2. Key design aspects

2.1. Lateral restraint spacing

EN 1993-1-1 (2005) defines a non-dimensional slenderness limit, or plateau length, $\bar{\lambda}_{LT} = 0.4$, below which, the effects of lateral torsional buckling can be ignored and the design buckling

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http://dx.doi.org/10.1016/j.tws.2014.11.018
0263-8231/

resistance moment of the member $M_{b,Rd}$ may be taken as the design bending resistance $M_{c,Rd}$ of the cross-section, assuming $\gamma_{M0}=\gamma_{M1}$. $\overline{\lambda}_{LT}$ is defined as

$$\overline{\lambda}_{LT}=\sqrt{\frac{W_y f_y}{M_{cr}}} \tag{1}$$

in which W_y is the major axis plastic section modulus for Class 1 and 2 cross-sections, the elastic section modulus for Class 3 cross-sections and an effective section modulus for Class 4 cross-sections, f_y is the material yield strength and M_{cr} is the elastic critical moment for lateral torsional buckling, which is a function of member length L. For a given set of cross-section and material properties and a fixed value of $\overline{\lambda}_{LT}$, Eq. (1) can be solved for L to define the maximum allowable spacing between fully effective lateral restraints before reductions in resistance for lateral torsional buckling are required. For members containing plastic hinges, stable lengths below which lateral torsional buckling can be ignored are given in Annex BB-1 of EN-1993-1-1 (2005).

2.2. Restraint forces

Lateral restraints must be of sufficient stiffness to restrict lateral buckling deformations at the point of restraint, whilst also being of sufficient strength to resist the forces generated as a result of the restraining action. In the elastic range, it can be shown that, for a perfect system, there is a threshold level of brace stiffness that causes a beam to buckle into the second mode (i.e. between the brace points rather than in an overall mode) – see Fig. 1 [3].

For a beam of length L experiencing a force N_{Ed} in the compression flange, EN 1993-1-1 states that the restraint system should be capable of resisting an equivalent stabilising force per unit length q_d:

$$q_d=\sum N_{Ed}\frac{8e_0+\delta_q}{L^2} \tag{2}$$

where the assumed initial imperfection amplitude of the restrained member, e_0, is defined as

$$e_0=\alpha_m L/500 \tag{3}$$

in which α_m is reduction factor used for restraining multiple members and δ_q is the lateral deflection of the restrained member into the restraints. Assuming an infinitely stiff restraint system, $\delta_q=0$, and Eq. (2) implies that a restraint must resist 1.6% of N_{Ed}. Eq. (2) is derived on the basis of elastic behaviour, but may also be applied when plasticity occurs in the restrained member, allowing for moments up to the full plastic bending capacity, M_{pl}, but not covering the demands of rotating plastic hinges. For non-rotating plastic hinges at M_{csm}, it has been previously established in [15] that no modifications are necessary to the current provisions of EN 1993-1-1. The additional deformation demands at rotating plastic hinges will place additional demands upon the bracing system. For

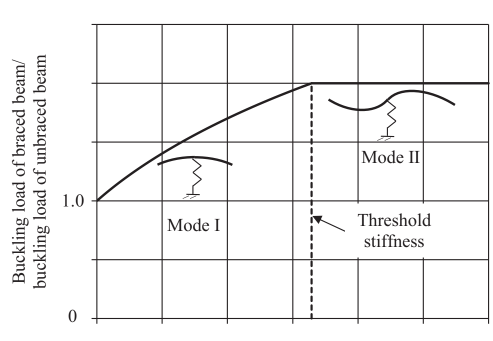

Fig. 1. Typical theoretical relationship between buckling load enhancement and restraint stiffness for a beam.

members that do contain rotated plastic hinges, the additional requirements in EN 1993-1-1 are as follows:

(i) At each plastic hinge location, the cross-section should have an effective lateral and torsional restraint, provided at both the tension and compression flanges.
(ii) The braces at the compression flange should be designed to resist a local force of at least 2.5% of N_{Ed}, where $N_{Ed}=M_{Ed}/h$ is the force in the compression flange, M_{Ed} is the moment in the beam at the plastic hinge location and h is the overall depth of the beam.

2.3. The continuous strength method (CSM)

The continuous strength method is a deformation-based design approach for steel elements that allows for the beneficial influence of strain-hardening. To date, design equations for the CSM have been developed for cross-section resistance in bending and compression [16]. The CSM bending resistance function $M_{csm,Rd}$, which applies for $\overline{\lambda}_p\le 0.68$ is defined as

$$M_{csm,Rd}=\frac{W_{pl}f_y}{\gamma_{M0}}\left(1+\frac{E_{sh}}{E}\frac{W_{el}}{W_{pl}}\left(\frac{\epsilon_{csm}}{\epsilon_y}-1\right)-\left(1-\frac{W_{el}}{W_{pl}}\right)\left(\frac{\epsilon_{csm}}{\epsilon_y}\right)^{-2}\right) \tag{4}$$

where E is the modulus of elasticity, E_{sh} is the strain-hardening slope taken equal to $E/100$ for structural steel, W_{el} and W_{pl} are the elastic and plastic section moduli and $\epsilon_{csm}/\epsilon_y$ is the strain ratio, defining the limiting strain in the cross-section as a multiple of the yield strain ϵ_y, and given by the following equation:

$$\frac{\epsilon_{csm}}{\epsilon_y}=\frac{0.25}{\overline{\lambda}_p^{3.6}} \quad \text{but} \quad \le 15 \tag{5}$$

in which $\overline{\lambda}_p$ is the local cross-section slenderness, given by the following equation:

$$\overline{\lambda}_p=\sqrt{\left(f_y/\sigma_{cr}\right)} \tag{6}$$

with σ_{cr} being the elastic buckling stress of the cross-section, or conservatively its most slender constituent plate element.

A key assumption of traditional plastic design is rigid-plastic behaviour, whereby upon attaining the plastic moment capacity no further increases in capacity occur with deformation and infinite rotations can be achieved. Introducing strain-hardening precludes the notion that plastic hinges may rotate at a constant moment, and with stocky sections (low $\overline{\lambda}_p$ values) significant increases in capacity beyond M_{pl} are possible [16]. Notwithstanding, the basic features of traditional plastic design (equilibrium, mechanism and yield) can be combined with the CSM by modifying the CSM moment capacity predictions at individual plastic hinge locations based upon relative deformation demands [17,16]. The procedure can be summarised in the following steps:

(i) Identify the locations of the plastic hinges and where necessary determine the critical collapse mechanism.
(ii) Using the theorem of virtual work, evaluate the rotations θ_i at each plastic hinge location i.
(iii) Based upon cross-section slenderness $\overline{\lambda}_p$, determine the deformation capacity $\epsilon_{csm}/\epsilon_y$ using Eq. (5).
(iv) For each plastic hinge, evaluate the ratio of deformation demand to deformation capacity, α_i, by using the following equation:

$$\alpha_i=\frac{\theta_i}{\left(\frac{\epsilon_{csm}}{\epsilon_y}\right)_i} \tag{7}$$