Thin-Walled Structures 86 (2015) 35–46

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Buckling analysis of thin-walled sections under localised loading using the semi-analytical finite strip method

Gregory J. Hancock*, Cao Hung Pham

School of Civil Engineering, The University of Sydney, NSW 2006, Australia

ARTICLE INFO

Article history:
Received 1 July 2014
Received in revised form
2 September 2014
Accepted 4 September 2014
Available online 25 October 2014

Keywords:
Thin-walled sections
Localised loading
Buckling analysis
Semi-analytical finite strip method
Spline finite strip method
Finite element method

ABSTRACT

Thin-walled sections under localised loading may lead to web crippling of the sections. This paper develops the Semi-Analytical Finite Strip Method (SAFSM) for thin-walled sections subject to localised loading to investigate web crippling phenomena. The method is benchmarked against analytical solutions, Finite Element Method (FEM) solutions, as well as Spline Finite Strip Method (SFSM) solutions. The paper summarises the SAFSM theory then applies it to the buckling of plates, and channel sections under localised loading. Multiple series terms in the longitudinal direction are used to compute the pre-buckling stresses in the plates and sections, and to perform the buckling analyses using these stresses. Solution convergence with increasing numbers of series terms is provided in the paper. The more localised the loading and buckling mode, the more series terms are required for accurate solutions. The loading cases of Interior One Flange (IOF) and Interior Two Flange (ITF) are investigated in this paper using simply supported boundary conditions.

1. Introduction

Thin-walled sections and plates under localised loading leading to plate buckling have been studied analytically for a long period mainly as part of investigations of web plates of sections at points of concentrated load. Two of the most comprehensive summaries of the work to date have been by Khan and Walker [1] where the buckling of plates subject to localised loading was investigated and Johansson and Lagerqwist [2] where the resistance of plate edges to localised loading is summarised. More recently, Natario et al. [3] have further developed these investigations for beams subjected to concentrated loads using Generalised Beam Theory (GBT). They benchmark GBT for plates, un-lipped channels and I-sections against the earlier research and the Shell Finite Element method (SFE). To date, the Finite Strip Method (FSM) of analysis developed by Cheung [4] does not appear to have been used for buckling studies under localised loading. As the FSM is used extensively in the Direct Strength Method (DSM) of design of cold-formed sections in the North American Specification NAS S100 [5] and the Australian/New Zealand Standard AS/NZS 4600 [6] it is important that the FSM of buckling analysis is extended to localised loading. This paper further develops the Semi-Analytical Finite Strip Method (SAFSM) for thin-walled sections subject to localised loading and benchmarks it against the Spline Finite Strip Method (SFSM) used previously by Pham and Hancock [7,8] for shear buckling problems and the Finite Element Method programme-ABAQUS/Standard (2008) version 6.8-2 [9].

Folded plate and finite strip theories for the buckling analysis of thin-walled sections and stiffened panels in longitudinal and transverse compression and shear have been developed since the mid-1960s. Two basic approaches were adopted. These are the exact solutions of Wittrick [10], and Williams and Wittrick [11], and the approximate solutions of Przemieniecki [12] and Plank and Wittrick [13] based on the finite strip method of analysis developed by Cheung [4]. Most of this research was applied to aeronautical structures where the longitudinal and transverse compression as well as shear is assumed constant as in the stiffened panels of aeroplane wings. Recently, Chu et al. [14] and Bui [15] have applied the SAFSM to the buckling of thin-walled sections under more general loading conditions so that multiple series terms are used to capture the modulation of the buckles that occur. These latter papers are restricted to bending of the sections and transverse compression and shear are not included. The application of the SAFSM to uniform shear of thin-walled sections has recently been applied by Hancock and Pham [16] where multiple series terms in the longitudinal direction are used to perform the buckling analyses. In the present paper, the method in Hancock and Pham [16] is extended to include the potential energy resulting from varying longitudinal, transverse and shear stresses. Multiple series terms in the longitudinal direction are used to compute the pre-buckling stresses in the plates and sections, and to perform the

* Correspondence to: School of Civil Engineering, Building J05, The University of Sydney, NSW 2006, Sydney, Australia. Tel.: +61 2 9351 2129; fax: +61 2 9351 3343.
E-mail addresses: gregory.hancock@sydney.edu.au (G.J. Hancock), caohung.pham@sydney.edu.au (C.H. Pham).

http://dx.doi.org/10.1016/j.tws.2014.09.017
0263-8231/

buckling analyses using these stresses. Solution convergence with increasing numbers of series terms is provided. The more localised the loading and hence buckling mode then the more series terms are required for accurate solutions especially for longer sections with concentrated loads.

2. Finite strip pre-buckling and buckling analyses

The finite strip method developed by Cheung [4] is similar to the finite element method in that approximate displacement functions are used to represent the plate flexural and membrane deformations. The theorem of minimum total potential energy is applied to compute the resulting stiffness equations. The SAFSM allows the deformations and stresses to be computed for any folded plate system satisfying the boundary conditions assumed. Normally, the sections are assumed simply supported at the ends so that the harmonic functions in the longitudinal direction are orthogonal thus allowing the different series terms to be uncoupled in the linear stiffness analysis. This produces considerable computational advantages for the SAFSM compared with the FEM. Longitudinal functions for other boundary conditions can be chosen which are also orthogonal as given by Cheung [4]. The resulting stiffness equations are summarised by

$$[K]\{\delta\} = \{W\} \tag{1}$$

where $[K]$ is the system stiffness matrix based on a strip subdivision of a thin-walled section as shown in Fig. 1, $\{\delta\}$ are the nodal line displacements of the strips in the global X,Y,Z axes, and $\{W\}$ is the load vector based on the constant nodal line loads F_X, F_Y and F_Z acting over a portion of the length as shown in Fig. 1. $[K]$ is based on the strain energy of the strip elements, and $\{W\}$ is based on the potential energy of the line loads. Its components are given in Appendix A4.

Eq. (1) can be solved for the nodal line displacements $\{\delta\}$ in the global X,Y,Z axes, and the flexural and membrane stresses $\{\sigma\}$ in the strips. These are pre-buckling displacements and stresses and are also described by harmonic functions.

Based on the pre-buckling membrane stresses $\{\sigma\}$, the stability equations given by Eq. (2) can be derived from the minimum total potential of the system undergoing buckling deformations. Since the buckling deformations also satisfy the simply supported boundary conditions, the same displacement functions are used for the buckling deformations as for the pre-buckling deformations.

$$([K]-\lambda[G])\{\delta\} = 0 \tag{2}$$

where $[G]$ is the system stability matrix and λ is the load factor against buckling. The paper concentrates the development of the stability matrix $[G]$ for strips under generalised membrane stresses resulting from line loading as shown in Fig. 1. The formulation of the stiffness matrix $[K]$ was given previously in detail in Cheung [4] and so its detailed derivation is not given in this paper since it is identical to Cheung [4]. All pre-buckling results in this paper are the same as would be derived using Cheung [4]. However, the flexural and membrane components of the stiffness matrix are given in Appendices A1 and A2 respectively for completeness and consistence of notation.

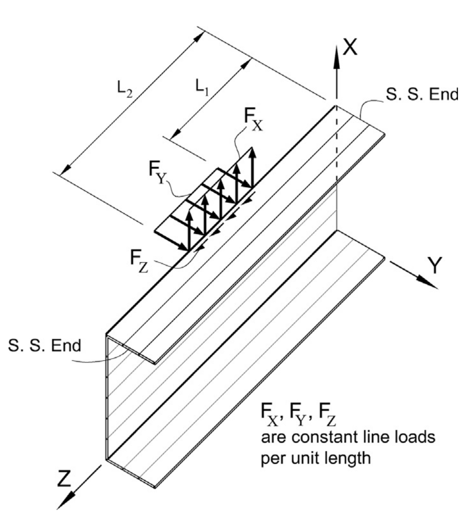

Fig. 1. Line loads on channel section showing global axes X,Y,Z.

3. Plate deformations

The plate flexural deformations (w) of a strip can be described by the summation over μ series terms as follows:

$$w = \sum_{m=1}^{\mu} f_{1m}(y)X_{1m}(x) \tag{3}$$

where the x-axis is in the longitudinal direction in the plane of the strip, the y-axis is in the transverse direction in the plane of the strip, and w is in the z-direction perpendicular to the strip as shown in Fig. 2.

The function $f_{1m}(y)$ for the mth series term is the transverse variation given by

$$f_{1m}(y) = \alpha_{1Fm} + \alpha_{2Fm}\left(\frac{y}{b}\right) + \alpha_{3Fm}\left(\frac{y}{b}\right)^2 + \alpha_{4Fm}\left(\frac{y}{b}\right)^3 \tag{4}$$

where the 4 polynomial coefficients α_{iFm} for the mth series term depend on the nodal line deformations of the strip. The term b is the width of the strip.

The function $X_{1m}(x)$ is the longitudinal variation of the mth series term and is given by

$$X_{1m}(x) = \sin\left(\frac{m\pi x}{L}\right) \tag{5}$$

where L is the length of the strip. The function $X_{1m}(x)$ satisfies the simply supported boundary conditions assumed in this paper. It is a useful function to analyse the IOF and ITF loading cases given in Refs. [5,6]. Other boundary conditions can be used in the SAFSM as set out in Cheung [4] for cases such as EOF and ETF in Refs. [5,6] but are not considered in this paper. They will be investigated in future papers.

The plate membrane deformations (u,v) in the (x,y) directions respectively can be described by the summation over μ series terms as

$$v = \sum_{m=1}^{\mu} f_{vm}(y)X_{1m}(x) \tag{6}$$

$$u = \sum_{m=1}^{\mu} f_{um}(y)X'_{1m}(x)\frac{L}{m\pi} \tag{7}$$

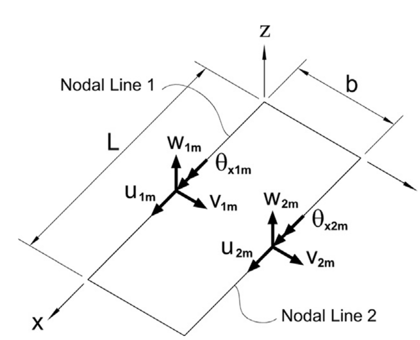

Fig. 2. Strip local axes X,Y,Z and nodal line deformations for the mth series term.