Thin-Walled Structures 86 (2015) 132–141

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Analytical simulation of nonlinear elastic–plastic average stress–average strain relationships for un-corroded/both-sides randomly corroded steel plates under uniaxial compression

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ARTICLE INFO

Article history:
Received 26 August 2012
Received in revised form
14 October 2014
Accepted 14 October 2014
Available online 6 November 2014

Keywords:
Imperfect steel plate
Both-sides random corrosion
Ultimate strength
Average stress–average strain relationship
Simulation

ABSTRACT

In this paper, an analytical simplified method for derivation of the average stress–average strain relationship of imperfect steel plates taking into account of both geometric and material nonlinearities is presented. The method utilizes the theory of elastic large deflection analysis of plates in the elastic region, and also the theory of rigid-perfectly plastic mechanism analysis of plates in the plastic region. The ultimate strength of the plate is predicted using an empirical formulation. The steel plates may be entirely un-corroded or both-sides randomly corroded. The algorithm can be easily implemented in methods for evaluation of ship hull girder ultimate strength as well as in the estimation of the ultimate capacity of offshore structures.

1. Introduction

In design of ships and offshore structures, it is essential to ensure that the structure has sufficient strength to sustain extreme loading situations. Such marine structures are mostly assembled of plates and plated elements. Thus, strength of plates and other plated elements is crucial for the overall structural capacity or in other words for the ultimate strength of the whole structure. For a thorough assessment of a structural design, for understanding possible improvements and to predict the consequences in the event of failure, an approximation of the value of ultimate strength is not sufficient. The complete behaviour, up to collapse and beyond, of the structure has to be simulated to gain insight into causes and effects of a structural failure.

For the analysis of large marine structures, an accurate and efficient approach is required to obtain results within a reasonable space of the time. Despite the enormous development in computer technology, elastic-plastic large deflection analyses with conventional finite element analysis (FEA) are too time-consuming for large structures. Therefore, a simplified method has to be employed to reduce the computational time and/or increase the size of the structural parts that can be analysed.

For cross sections of ships in bending, methods to obtain the moment-curvature relationship, Fig. 1, considering the collapse of parts of the cross section have been developed. One of the most known methods is the Smith's method [1,2], in which the ship cross-section is divided into small elements each of which is composed of plates without/with stiffener. Average stress–average strain relationships of all elements are derived before the analysis of the whole cross-section progresses as follows: curvature is applied incrementally about the instantaneous neutral axis, the strain of each element is calculated, the corresponding stress is taken from the stress–strain curves previously derived, and the corresponding moments is obtained by integration over the cross-section, Fig. 2. FEA is usually applied in order to derive the average stress–average strain relationships of plate and stiffened plate elements. Application of FEA in derivation of average stress–average strain relationships of the plated elements in the cross-section of a ship hull girder or any other box-shape structure would mean spending a considerable amount of cost and time. Therefore, it is felt that there is a need to develop or propose a simplified method in order to perform such calculations. These calculations create a significant step in progressive collapse analysis of marine structures.

Analytical method proposed in this paper, is one suitable framework for implementing a general approach to collapse analysis, since it leads to the reduction of solution process either in time or in cost. Combining the theory of elastic large deflection analysis with rigid-plastic mechanism analysis, a simple formulation is expressed in order to derive average stress–average strain relationships of plates. The accuracy of the method or formulation is verified against the FEA obtained results. Employing such a formulation, the ultimate strength

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http://dx.doi.org/10.1016/j.tws.2014.10.012
0263-8231/

Notations

AR	aspect ratio of the plate
a	plate length
b	plate breadth
t	thickness of plate in un-corroded condition
t_{eq}	effective thickness of plate in corroded condition
t_P	thickness function of plate in un-corroded condition
E	Young modulus of material
ν	Poisson's ratio of material
m	number of half-waves in longitudinal direction
m_0, m_{45}, m_{90}	some constants in the rigid-plastic mechanism analysis relationships
n	number of half-waves in transverse direction
β	plate slenderness
F	Airy's stress function
μ	mean corrosion depth
S	standard deviation of random thickness variations
n_y	number of years of exposure
d_w	uniform reduction in thickness
r_1, r_2	random numbers corresponding to the corroded surfaces of the plate
z_{UpSRF}	Z-coordinate of the upper surface of the plate
z_{LowSRF}	Z-coordinate of the lower surface of the plate
U_x	displacement along x-axis
U_y	displacement along y-axis
U_z	displacement along z-axis
A_{0ij}	coefficients in initial deflection function
w_0	initial deflection function
$w_{0\,\max}$	maximum magnitude of initial deflection
w_e	elastic deflection
w_p	plastic deflection
ε	strain
ε_Y	material yield strain
σ	stress
σ_Y	material yield stress
σ_U	material ultimate stress
σ_{Ult}	ultimate compressive strength of the plate
$_r$, $_{r-1}$	rth and $(r-1)$th values of the parameter

evaluation of ships and offshore structures is possible in a very short time with reasonable accuracy and cost.

2. General assumptions

The longitudinal stiffening system is usually employed in large ships at their mid-length part, especially in the deck and bottom structures. If an extreme bending moment acts on a hull girder, the most possible collapse mode may be the overall collapse of stiffened panel after local collapse of individual plate elements between stiffeners.

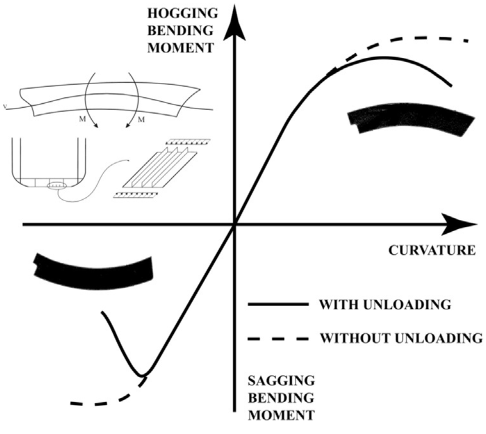

Fig. 1. Typical moment–curvature relationship.

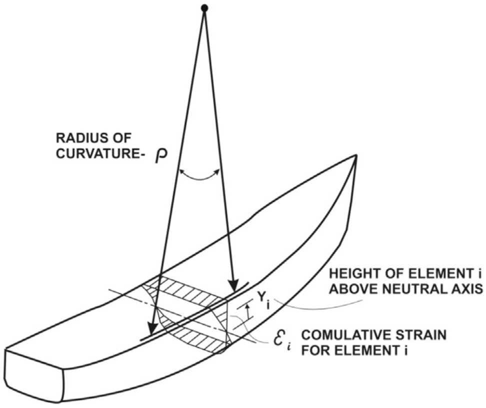

Fig. 2. Ship hull girder bending concept (Smith's method).

The following assumptions are made in the derivation of the average stress–average strain relationships of the plate elements:

1. Attached plating between longitudinal stiffeners behaves as an isolated plate.
2. The material is assumed to be elastic-perfectly plastic.

Average stress–average strain relationships of the isolated plates are derived combining the results of elastic large deflection analysis and rigid plastic mechanism analysis.

3. Average stress–average strain relationship of un-corroded plates

3.1. Welding induced initial deflections

In a thin-walled plated structure, the initial deflection of a local plate panel is produced by the fillet welding between plate and stiffeners. The resulting typical shape of initial deflection is the so-called *thin-horse mode* that deflects in the same direction in adjacent spans or bays. An example of initial deflection of the inner-bottom plating of an existing Handy-sized Bulk Carrier, measured by Yao et al. [3], is illustrated in Fig. 3. As shown, the rectangular plate panels have an almost symmetric mode of initial deflection across the bays and spans. The usual assumption of asymmetric modes of initial deflection based on the linear elastic

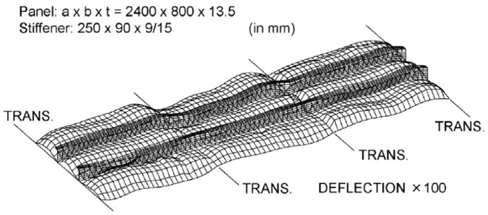

Fig. 3. Real distribution of initial deflection or so-called thin-horse mode initial deflection [3].