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Buckling analysis of thin-walled functionally graded sandwich box beams

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ABSTRACT

Buckling analysis of thin-walled functionally graded (FG) sandwich box beams is investigated. Material properties of the beam are assumed to be graded through the wall thickness. The Euler-Bernoully beam theory for bending and the Vlasov theory for torsion are applied. The non-linear stability analysis is performed in framework of updated Lagrangian formulation. In order to insure the geometric potential of semitangental type for internal bending and torsion moments, the non-linear displacement field of thin-walled cross-section is adopted. Numerical results are obtained for FG sandwich box beams with simply–supported, clamped–free and clamped–clamped boundary conditions to investigate effects of the power-law index and skin-core-skin thickness ratios on the critical buckling loads and post-buckling responses. Numerical results show that the above-mentioned effects play very important role on the buckling analysis of sandwich box beams.

1. Introduction

In recent years, there is a rapid increase in the use of functionally graded (FG) structures. Thin-walled FG beams have practical interest and future potential particularly in aerospace and mechanical applications due to high strength-to-weight ratio. Many papers have been devoted to study bending, vibration and buckling of FG and FG sandwich beams and only some of them are cited here [1–13]. In these papers, different theories (classical beam theory, first-order beam theory and higher-order beam theory) and various material distribution laws of FG beams have been introduced. However, there are quite a few papers which mainly studied dynamics of FG thin-walled box beams. Librescu et al. [14] studied instability, vibration analysis along with the effects of temperature gradients and volume fraction of FG thin-walled beams. Piovan and Machado [15] adopted a second-order nonlinear displacement field in order to study the dynamic stability of simply–supported FG box beams under an axial external force. Based on the first-order shear deformation theory, Ziane et al. [16] investigated the free vibration of FG box beams by using the formulation of an exact dynamic stiffness matrix. Carvalho et al. [17] studied the nonlinear nonplanar vibration of a clamped–free sler FG box beam. Mashat et al. [18] used Carrera Unified Formulation to perform vibration of thin- and thick-walled FG box beams.

In this paper, which is an extension of previous work [19], buckling analysis of FG sandwich box beams is presented. Material properties of the beam are assumed to be graded across the wall thickness.

The model is based on assumptions of large displacements but small strains, the Euler-Bernoully beam theory for bending and the Vlasov theory for torsion. The thin-walled beam members are supposed to be straight and prismatic. External loads are assumed to be static and conservative. In order to perform non-linear stability analysis in load deflection manner, the updated Lagrangian (UL) incremental descriptions is applied. The non-linear cross section displacement field which accounts for the second order displacement terms due to large rotations is implemented. The generalized displacement control method is employed in terms of the incremental-iterative solution scheme. Updating of nodal orientations at the end of the each iteration is performed using the transformation rule which applies for semitangental incremental rotations, while the force recovering is performed according to the conventional approach (CA).

Numerical results are obtained for sandwich box beams with simply–supported, clamped free and clamped–clamped boundary conditions to investigate effects of the power-law index and skin-core-skin thickness ratios on the critical buckling loads and post-buckling respones. Numerical results show that the abovementioned effects play very important role on the buckling analysis of sandwich box beams.

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2. Basic consideration

To derive the finite element model of a thin-walled FG box beam, the following assumptions are made:

(1) The contour of the thin wall does not deform in its own plane.
(2) The linear shear strain $\overline{\gamma}_{zs}$ of the middle surface is to have the same distribution in the contour direction as it does in the St. Venant torsion in each element.
(3) The bending shear deformation are neglected.
(4) The local buckling as well as the distortional buckling are not considered.

2.1. Beam kinematics

In this paper, two sets of coordinate systems, which are mutually interrelated, are used. The first coordinate system is Cartesian coordinate system (z, x, y), for which z-axis coincides with the beam axis passing through the centroid O of each cross-section, while the x- and y-axes are the principal inertial axes of the cross-section taken along the width and height of the beam. The second coordinate system is contour coordinate (z, n, s) as shown in Fig. 1, wherein coordinate z coincident with beam z-axis, the coordinate s is measured along the tangent of the middle surface in a counter-clockwise direction, while n is the coordinate perpicular to s. Incremental displacement measures of a cross-section are defined as

$$w_o = w_o(z), \quad u_o = u_o(z), \quad v_o = v_o(z), \quad \varphi_z = \varphi_z(z),$$
$$\varphi_x = -v_o' = \varphi_x(z), \quad \varphi_y = u_o' = \varphi_z(z), \quad \theta = -\varphi_z' = \theta(z) \tag{1}$$

where w_o, u_o and v_o are the rigid-body translations of the cross-section associated with the centroid in the z-, x- and y-directions, respectively; φ_z, φ_x and φ_y are the rigid-body rotations about the z-, x- and y-axis, respectively; θ is a parameter defining the warping of the cross-section. The superscript 'prime' indicates the derivative with respect to z.

If rotations are small, the incremental displacement field of a thin-walled cross-section contains only the first-order displacement terms [20]:

$$u_z = w_o - yv_o' - xu_o' - \omega\varphi_z',$$
$$u_x = u_o - y\varphi_z,$$
$$u_y = v_o + x\varphi_z \tag{2}$$

in which u_z, u_x and u_y are the linear or first-order displacement increments of an arbitrary point on the cross-section defined by the position coordinates x and y and the warping function $\omega(x, y)$. If the assumption of small rotations is not valid, then the second-order displacement increments:

$$\tilde{u}_z = 0.5\left[\varphi_z\varphi_x x + \varphi_z\varphi_y y\right],$$

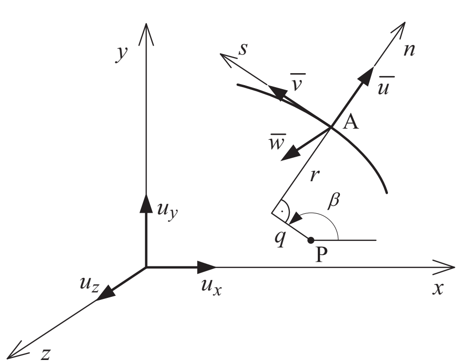

Fig. 1. Contour displacements with respect to beam displacements.

$$\tilde{u}_x = 0.5\left[\varphi_x\varphi_y y - \left(\varphi_z^2 + \varphi_y^2\right)x\right],$$
$$\tilde{u}_y = 0.5\left[\varphi_x\varphi_y x - (\varphi_z^2 + \varphi_x^2)y\right] \tag{3}$$

due to large rotations should be added to those from Eq. (2).

The strain tensor components, corresponding to nonlinear displacement field, are:

$$\varepsilon_{ij} = \frac{1}{2}\left[(u_i + \tilde{u}_i)_{,j} + \left(u_j + \tilde{u}_j\right)_{,i} + (u_k + \tilde{u}_k)_{,i}(u_k + \tilde{u}_k)_{,j}\right] \cong e_{ij} + \eta_{ij} + \tilde{e}_{ij} \tag{4}$$

$$2e_{ij} = u_{i,j} + u_{j,i}, \quad 2\eta_{ij} = u_{k,i}u_{k,j}, \quad 2\tilde{e}_{ij} = \tilde{u}_{i,j} + \tilde{u}_{j,i} \tag{5}$$

The last term $\tilde{e}_{ij}$ contains the second-order displacements due to the large rotations.

2.2. Contour displacements

The contour mid-line displacements are $\overline{w}, \overline{u}, \overline{v}$, while the out of mid-line displacement components are defined as:

$$w(z,s,n) = \overline{w} - n\frac{\partial \overline{u}}{\partial z}; \quad v(z,s,n) = \overline{v} - n\frac{\partial \overline{u}}{\partial s}; \quad u(z,s,n) = \overline{u}. \tag{6}$$

Beam to contour displacement relation can be given as:

$$\overline{w}^L = u_z(z,s,n), \overline{w}^{NL} = \tilde{u}_z(z,s,n);$$
$$\overline{v}^L = u_x(z,s,n)\cos\beta + u_y(z,s,n)\sin\beta,$$
$$\overline{v}^{NL} = \tilde{u}_x(z,s,n)\cos\beta + \tilde{u}_y(z,s,n)\sin\beta;$$
$$\overline{u}^L = u_x(z,s,n)\sin\beta - u_y(z,s,n)\cos\beta,$$
$$\overline{u}^{NL} = \tilde{u}_x(z,s,n)\sin\beta - \tilde{u}_y(z,s,n)\cos\beta \tag{7}$$

where indexes L and NL indicates linear and nonlinear parts, respectively.

Out of mid-line displacements can also be separate into linear and non-linear components:

$$w^L(z,s,n) = \overline{w}^L - n\frac{\partial \overline{u}^L}{\partial z}; \quad v^L(z,s,n) = \overline{v}^L - n\frac{\partial \overline{u}^L}{\partial s}; \quad u^L(z,s,n) = \overline{u}^L \tag{8}$$

$$w^{NL}(z,s,n) = \overline{w}^{NL} - n\frac{\partial \overline{u}^{NL}}{\partial z}; \quad v^{NL}(z,s,n) = \overline{v}^{NL} - n\frac{\partial \overline{u}^{NL}}{\partial s};$$
$$u^{NL}(z,s,n) = \overline{u}^{NL} \tag{9}$$

The only non-zero strain components, according to Bernoulli hypothesis, are:

$$e_{zz} = \frac{\partial w^L}{\partial z}; e_{zs} = \frac{\partial w^L}{\partial s} + \frac{\partial v^L}{\partial z}; \tag{10}$$

$$\eta_{zz} = \frac{1}{2}\left[\left(\frac{\partial w^L}{\partial z}\right)^2 + \left(\frac{\partial u^L}{\partial z}\right)^2 + \left(\frac{\partial v^L}{\partial z}\right)^2\right];$$
$$\eta_{zs} = \frac{\partial w^L}{\partial z}\frac{\partial w^L}{\partial s} + \frac{\partial u^L}{\partial z}\frac{\partial u^L}{\partial s} + \frac{\partial v^L}{\partial z}\frac{\partial v^L}{\partial s}; \tag{11}$$

$$\tilde{e}_{zz} = \frac{\partial w^{NL}}{\partial z}; \quad \tilde{e}_{zs} = \frac{\partial w^{NL}}{\partial s} + \frac{\partial v^{NL}}{\partial z} \tag{12}$$

where e_{ij} and η_{ij} are linear and non-linear strains with respect to linear displacements components, while $\tilde{e}_{ij}$ are linear strains with respect to nonlinear displacements.

By putting Eq. (8) into Eq. (10) follows:

$$e_{zz} = \frac{\partial w^L}{\partial z} = \frac{\partial \overline{w}^L}{\partial z} - n\frac{\partial^2 \overline{u}^L}{\partial z^2} = \overline{\varepsilon}_z{}^L + n\kappa_z{}^L \tag{13}$$

$$e_{zs} = \frac{\partial w^L}{\partial s} + \frac{\partial v^L}{\partial z} = \frac{\partial \overline{w}^L}{\partial s} + \frac{\partial \overline{v}^L}{\partial z} - 2n\frac{\partial^2 \overline{u}^L}{\partial s \partial z}\frac{\partial w^L}{\partial s} + \frac{\partial v^L}{\partial z} = \overline{\gamma}_{zs}^L - 2n\frac{\partial^2 \overline{u}^L}{\partial s \partial z}$$
$$= \overline{\gamma}_{zs}^L + n\kappa_{zs}{}^L. \tag{14}$$

The middle surface shear strain $\overline{\gamma}_{zs}^L$, in accordance with second assumption, will be: