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A semi-analytical approach for linear and non-linear analysis of unstiffened laminated composite cylinders and cones under axial, torsion and pressure loads

Saullo G.P. Castro [a,b,c,*], Christian Mittelstedt [d], Francisco A.C. Monteiro [e], Mariano A. Arbelo [a], Richard Degenhardt [a,f], Gerhard Ziegmann [c]

[a] PFH, Private University of Applied Sciences Göttingen, Airbus-Strasse 6, 21684 Stade, Germany
[b] Embraer, Brazilian Aerospace Company, 12227-901 São José dos Campos, SP, Brazil
[c] Clausthal University of Technology, Institute for Polymaterials and Plastic Engineering, Agricolastr. 6, 38678 Clausthal-Zellerfeld, Germany
[d] SOGETI High Tech GmbH, Dept. ANST – Analytical and Numerical Simulation & Testing, Hein-Sass-Weg 30, D-21129 Hamburg, Germany
[e] ITA, Technological Institute of Aeronautics, LME – Laboratory of Structural Modelling, 12228-900 São José dos Campos, SP, Brazil
[f] DLR, Institute of Composite Structures and Adaptive Systems, Lilienthalplatz 7, 38108 Braunschweig, Germany

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ABSTRACT

A semi-analytical model for the non-linear analysis of simply supported, unstiffened laminated composite cylinders and cones using the Ritz method and the Classical Laminated Plate Theory is proposed. A matrix notation is used to formulate the problem using Donnell's and Sanders' non-linear equations. The approximation functions proposed are capable to simulate the elephant's foot effect, a common phenomenon and a common failure mode for cylindrical and conical structures under axial compression. Axial, torsion and pressure loads can be applied individually or combined, and solutions for linear static, linear buckling and non-linear buckling analyses are presented and verified using a commercial finite element software. The presented non-linear buckling analyses used perturbation loads to create the initial geometric imperfections, showing the capability of the method for arbitrary imperfection patterns. The linear stiffness matrices are integrated analytically and for the conical structures an approximation is proposed to overcome the non-integrable expressions.

1. Introduction

The structural behavior of cylindrical structures has been a topic for research in the past 120 years. Southwell [1] is one of the pioneering authors presenting equations for the buckling of thin-walled isotropic cylinders, already recognizing that his theory could not be applied in real cases where geometric imperfections and load asymmetries took place based on the observed discrepancies between theory and test results. The first developments of non-linear buckling equations focused on isotropic materials and the increasing application of composite structures, especially for aerospace and space structures, motivated the development of theories for orthotropic materials. Tennyson [2] presents a thorough review about the first studies with orthotropic materials, all of them constraining the equations for symmetric or anti-symmetric laminates [3]. Simitses et al. [3] are among the first authors developing non-linear buckling equations for general laminated composite cylinders.

Concerning the different non-linear approximations used for the kinematic relations, Simitses et al. [4] presents a comparison between the Donnell [5] and the Sanders [6] approximations for the buckling of axially compressed orthotropic cylinders under axisymmetric imperfections, and the general trend observed by the authors it that Donnell's equations can overestimate the buckling load, especially for thinner and longer cylinders. Goldfeld et al. [7] extended the study of Simitses et al. [4] to isotropic conical shells and included the terms required for the Timoshenko and Gere's kinematic approximation [8], concluding that Sanders' approximation already gives an accuracy comparable to Timoshenko and Gere's approximation, and supporting the observation that the more accurate non-linear equations results in a lower buckling load predictions. In 2007, Goldfeld [9] used a model with variable thickness for laminated composite cones and verified that the imperfection sensitivity is not reduced for a wider semi-vertex angle and observed that the most accurate non-linear equations result in lower buckling loads and less imperfection sensitivity.

In the approaches of Simitses et al. [3,4] and of Goldfeld et al. [7,9] only symmetric and axisymmetric imperfection patterns can be used, and more general imperfection patterns cannot be used, such as those obtained using perturbation loads or geometric imperfections from

* Corresponding author. Tel.: +55 12 98113 3855.
E-mail address: castrosaullo@gmail.com (S.G.P. Castro).

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advanced data acquisition systems [10]. Such general imperfection patterns are not necessarily symmetric and can only be represented by appropriate Fourier series such as the half-cosine function proposed by Arbocz [11].

Geier and Singh [12] are among the first authors to present the linear buckling formulations for laminated composite cylinders and panels, developing equations for thin and moderately thick shells, where the thick shell formulation is similar to Sanders' approximation herein adopted. The authors assumed an orthotropic laminate where the terms A_{16}, A_{26}, B_{16}, B_{26}, D_{16}, D_{26} and the corresponding symmetric counterparts in the laminate constitutive matrix are set to zero. This assumption is required in order to allow the derivation of analytical solutions for the linear buckling problem, but as a side effect any shear–tension, torsion–tension and torsion–bending coupling is ignored, resulting in buckling modes which resemble those of isotropic structures, i.e. without a torsion-like pattern. More recently Shadmehri [13] proposed a semi-analytical approach for the linear buckling of laminated composite cones where the approximation functions also do not allow the referred torsion-like patterns since it does not contain a complete Fourier series for the circumferential coordinate, resulting always in a null normal displacement ($w=0$) at $\theta=0$ and $\theta=\pi$ (cf. Fig. 1).

The present study proposes a semi-analytical model based on the Ritz method with the aim to fill a literature gap for unstiffened cylindrical and conical shells, developing a formulation that is capable to predict the linear and non-linear behavior of these structures under any arbitrary configuration of geometric imperfections. This paper focus on imperfections created using a single perturbation load, which are relevant for the Single Perturbation Load Approach (SPLA), proposed by Hühne and collaborators [14–16], which consists of new design approach for less conservative estimations of knock-down factors commonly applied in space structures. Such imperfection pattern has shown to produce less conservative knock-downs than axisymmetric or linear buckling mode-shaped imperfection patterns [17].

Besides the perturbation loads, axial compression, torsion and internal pressure are covered in the formulation herein developed. The shells are modeled using the Classical Laminated Plate Theory (CLPT), where a zero transverse shear state is assumed [18]. Donnell's and Sanders' equations are compared in order to evaluate whether the increased accuracy obtained using Sanders' justifies the higher computational cost associated. The non-linear formulation is derived in matrix form making it straightforward to construct the systems of equations from which the linear and non-linear static problems can be formulated. The matrices defined in the non-linear formulation are directly applied to obtain the non-linear eigenvalue problem using the

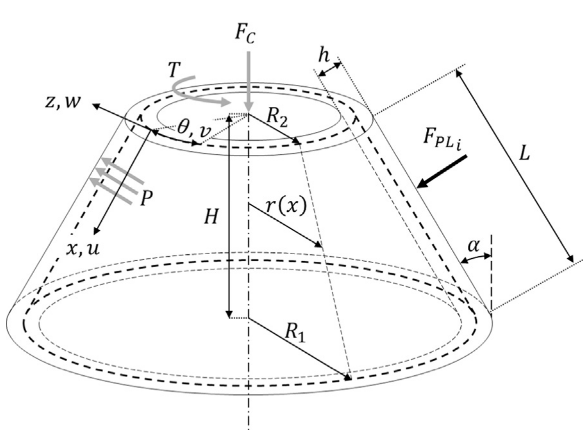

Fig. 1. Cone/cylinder coordinate system and geometric variables.

neutral equilibrium criterion, which is then specialized to the linear case in order to obtain the linear buckling equations.

In the context of linear and non-linear static analyses the proposed approximation functions can capture the so called “elephant's foot” effect, which consists of a high variation of the normal displacements close to the edges due to a non-uniform increase of the cone/cylinder radius during axial compression. The elephant's foot may cause a failure mode that governs the design of many practical structures [19], and has also been observed in the finite element models used to predict the non-linear buckling load of laminated composite cylinders [17,20].

2. Formulation of the different analysis types

In this section a set of non-linear equations for the static analysis of a conical/cylindrical shell is obtained using the Ritz method. The quantities described in all equations are expressed in terms of the initial, undeformed state (analogous to total Lagrangian formulation used in finite elements [21]). In Fig. 1 the adopted coordinate system and the variables defining the geometry and the displacement field are shown. The applied loads are defined by an axially compressive force F_C, a torsion load T, a pressure load P and perturbation loads F_{PL_i}. Note that the perturbation loads are applied normally to the shell surface. When only one perturbation load is applied, it will be called a single perturbation load (SPL). The cone becomes a cylinder when the semi-vertex angle α is set to zero. The radius $r(x)$ will be simply referred to as r for the sake of brevity and is calculated using Eq. (1).

$$r(x)=r=R_2+x\ \sin\ \alpha \tag{1}$$

Defining the total potential energy of the system of Fig. 1 as Π, the strain energy (internal energy) as U and the energy due to the external forces (external energy) as V, the equilibrium can be stated by the stationary condition [18,22]

$$\delta\Pi=\delta U+\delta V=0 \tag{2}$$

where

$$\delta U=\int_{\mathcal{V}}\{\sigma\}^T\{\delta\varepsilon\}d\mathcal{V} \tag{3}$$

and:

$$\delta V=-\{F_{ext}\}^T\{\delta c\} \tag{4}$$

with $\{F_{ext}\}$ calculated as detailed in Section 3. Using the Equivalent Single-Layer (ESL) kinematic assumptions [18] the integration of Eq. (3) can be approximated by an integration over the shell surface

$$\delta U=\int_{\mathcal{V}}\{\sigma\}^T\{\delta\varepsilon\}d\mathcal{V}=\iiint_{x\theta z}\{\sigma\}\{\sigma\}^T\{\delta\varepsilon\}rd\theta dx$$
$$=\iint_{x\theta}\{N\}^T\{\delta\varepsilon\}rd\theta dx \tag{5}$$

where $\{N\}$ is the vector of the distributed forces and moments acting on the shell which in the current study correlates to the strain vector using the Classical Laminated Plate Theory (CLPT) [18]. The components of the strain and distributed force vectors for the CLPT are

$$\{\varepsilon\}^T=\left\{\varepsilon_{xx}^{(0)}\quad\varepsilon_{\theta\theta}^{(0)}\quad\gamma_{x\theta}^{(0)}\quad\varepsilon_{xx}^{(1)}\quad\varepsilon_{\theta\theta}^{(1)}\quad\gamma_{x\theta}^{(1)}\right\}$$
$$\{N\}^T=\left\{N_{xx}\quad N_{\theta\theta}\quad N_{x\theta}\quad M_{xx}\quad M_{\theta\theta}\quad M_{x\theta}\right\}. \tag{6}$$

The material constitutive relations are assumed to be linear, being represented as

$$\{N\}=[F]\{\varepsilon\}. \tag{7}$$