Thin-Walled Structures 90 (2015) 95–106

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Parametric and non-parametric probabilistic approaches in the mechanics of thin-walled composite curved beams

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ARTICLE INFO

Article history:
Received 8 September 2014
Received in revised form
20 December 2014
Accepted 21 December 2014
Available online 2 February 2015

Keywords:
Uncertainties quantification
Composite curved beams
Dynamics
Flexible structures

ABSTRACT

In this paper we perform a quantification of the uncertainty propagation of the dynamics of slender initially curved structures constructed with fiber reinforced composite materials. Depending on the manufacturing process, composite materials may have deviations with respect to the expected response, often called nominal response in a deterministic sense. The manufacturing aspects lead to uncertainty in the structural response associated with constituent proportions, material and/or geometric parameters among others. Another aspect of uncertainty that can be sensitive in composite structures is the mathematical model that represents the mechanics of the structural member, that is: the assumptions and type of hypotheses invoked reflect the most relevant aspects of the physics of a structure, however in some circumstances these hypotheses are not enough, and cannot represent properly the mechanics of the structure. Uncertainties should be considered in a structural system in order to improve the predictability of a given modeling scheme. There are two approaches to evaluate the propagation of uncertainties in structural models: the parametric probabilistic approach and the non-parametric probabilistic approach. In the parametric, one quantifies the uncertainty of given parameters (such as variation of the angles of fiber reinforcement and material constituents) by associating random variables to them. In the non-parametric, the propagation of uncertainty is quantified by considering uncertain the matrices of the whole system. In this study a shear deformable model of composite curved thin-walled beams is employed as the mean or expected model. The probabilistic model is constructed by adopting random variables for the uncertain entities (parameters or matrices) of the model. The probability density functions of the random entities are derived appealing to the maximum entropy principle under given constraints. Once the probabilistic model is discretized in the context of the finite element method, the Monte Carlo method is employed to perform the simulations. Then the statistics of the simulations is evaluated and the parametric and non-parametric approaches are compared.

1. Introduction

Composite materials have such a number of interesting features that impel their use in different industrial devices. Examples of these features are high strength and stiffness properties together with a low weight, good corrosion resistance, enhanced fatigue life, low thermal expansion properties among others [1]. The very low machining cost for complex structures is the other important feature of composite materials [2]. Slender composite structures that can be analyzed by means of curved beam models are present in many applications such as bridge segments, machine parts: such as leaf springs of sport cars or blades of turbo-propellers, among others.

The development of theoretical and computational methods for dynamic and static analysis of slender thin-walled composite structures is growing continuously since the early eighties. Thus, one of the first consistent studies about thin-walled composite-beams was introduced by Baud and Tzeng [3], who developed a beam theory to analyze fiber-reinforced members featuring open cross-sections with symmetric laminates invoking Vlasov's hypotheses. Afterwards, Bauchau [4] incorporated some aspects of shear flexibility in the analysis of thin-walled composite beams. Models of fiber reinforced composite beams that are based on Vlasov or Bauld and Tzeng's ideas [3] normally over-predict the values of natural frequencies and consequently the dynamic patterns, specially in the case of shorter beams. In the 1990s, many new models of composite beams were introduced, in which

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http://dx.doi.org/10.1016/j.tws.2014.12.018
0263-8231/

shear-flexibility as well as warping effects due to non-uniform twisting were incorporated. These models were based on new theories for micro/macrostructures of composite materials, new modeling schemes including selective warping and second-order displacements, etc. The research of Wu and Sun [5], Librescu and coworkers [6,7], Kim et al. [8] and Cesnik et al. [9] are just a few examples of the most representative works in the modeling of composite beams with thin or thick walled cross-sections; however most of them were devoted to closed cross-sections as basic approaches for the analysis of helicopter blades. More recently Cortínez and Piovan [10] developed a theory of thin walled composite beams accounting for full shear flexibility (i.e. shear deformation due to bending as well as due to warping related to non-uniform twisting). The scopes and limits of the previous full shear flexible modeling conception were extended [11] by incorporating elastic couplings and the evaluation of general dynamic problems for straight beams and for curved thin-walled composite beams [12]. The model employed in this study was conceived in order to take into account the effects of shear deformability that are mandatory in the mechanics of thin-walled structures specially if they are constructed with fiber reinforced composite materials [6,8,10,12].

The behavior of composite structures under typical service in civil, aeronautical, aero-spatial or mechanical devices, is constrained to a number of factors that are stochastic in essence [13,14]. Many researchers have focused their attention in the evaluation of the stochastic response of composite structures since the middle 1990s [15,16]. Moreover, there is an increasing interest to quantify the propagation of uncertainty in the mechanics of composite materials at the microscale level [13] or for failure analysis [17]. The uncertainty involved in the material properties of the composites can be considered as random fields [18,19] among others. However, there are other ways for studying the dynamic response due to uncertainties in composite structures, for example by associating random variables to given entities that define a structural dynamic model. Effectively, when the parameters, such as material properties or reinforcement angles, are considered uncertain, the methodology for studying the uncertainty is called parametric probabilistic approach (PPA). However if the model as a whole is uncertain, the class of uncertainty is called systemic uncertainty. In order to analyze this type of uncertainty there are various approaches, one of them is the so-called non-parametric probabilistic approach (NPPA). The NPPA implies the introduction of random matrix variables. This approach was formulated by Soize [20] and employed in a variety of structural problems [21–23].

In this paper, the PPA and NPPA are applied in order to evaluate the uncertainty propagation in the dynamic response of naturally curved composite thin-walled beams. The theory for curved composite structures introduced by Piovan and Cortínez [12] is briefly revisited and employed as the nominal response or deterministic model in order to compare and quantify the uncertainty propagation of the stochastic approach. The solution of the dynamics equations is approximated in the context of the finite element method. For the PPA case, the parameters corresponding to elastic properties are considered uncertain. For the NPPA the stiffness matrix and the damping matrix are considered uncertain. This is due to the evidence gathered in other work of the authors [23] in which the elastic properties, and hence the stiffness matrix, are the main focus of uncertainty propagation in dynamics of composite thin-walled straight beams. To construct the probabilistic models, the probability density functions associated with the random variables are constructed appealing to the maximum entropy principle [24,25]. This principle uses the available information of the random entities to construct their probability density functions such that the entropy, in the sense of Shannon [26], is maximum. The use of this scheme allows the maximum possible propagation of the uncertainty according to the available information about the random variables.

The paper is organized as follows: after the introductory section where the state-of-the-art in modeling curved thin-walled composite beams is summarized, the deterministic/mean model and its finite element discretization are briefly described, then the probabilistic approach is constructed. The parametric and the non-parametric approaches are described for this problem and the subsequent section contains the computational studies, the analysis of the uncertainty propagation in the dynamics of thin-walled composite curved beams and finally concluding remarks are outlined.

2. Deterministic model

2.1. Brief description of the curved beam model

Fig. 1 shows a basic sketch of the structural component, in which it is possible to see the basic dimensions and the reference points ***C*** and ***A***. The principal reference point ***C*** is located at the geometric center of the cross-section, the x-direction is tangent to the curved axis of the beam, and y and z are the axes of the cross section, but not necessarily the principal axes of inertia. The secondary reference system, located at ***A***, is used to describe shell stresses and strains. The curved axis of the beam, that has constant radius R, is contained in the plane Ξ. The curved beam has an opening angle β and a circumferential length $L = R\beta$. The deterministic model of the present study is based on the following assumptions [11,12]:

1. The cross-section contour is rigid in its own plane (i.e. plane YZ).
2. The radius of curvature at any point of the shell is neglected.
3. The warping function is normalized with respect to the principal reference point ***C***.
4. A general laminate stacking sequence for a composite material is considered.
5. The material density is considered constant along the curved axis beam.
6. Stress and strain components are defined according to the secondary reference system located in ***A***.
7. The most representative stresses are σ_{xx}, σ_{xs} and σ_{xn}; and the most representative strain and curvature components are ϵ_{xx}, γ_{xs}, γ_{xn}, κ_{xx} and κ_{xs}.
8. The model is derived in the framework of linear elasticity.

Employing assumptions (1)–(7) one can derive the displacement field of the point ***B*** [12], which can be presented as follows:

$$\tilde{\mathbf{U}}_{\boldsymbol{B}} = \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix} = \begin{Bmatrix} u_{xc} - \omega \Phi_w \\ u_{yc} \\ u_{zc} \end{Bmatrix} + \begin{bmatrix} 0 & -\Phi_3 & \Phi_2 \\ \Phi_3 & 0 & -\Phi_1 \\ -\Phi_2 & \Phi_1 & 0 \end{bmatrix} \begin{Bmatrix} 0 \\ y \\ z \end{Bmatrix}, \quad (1)$$

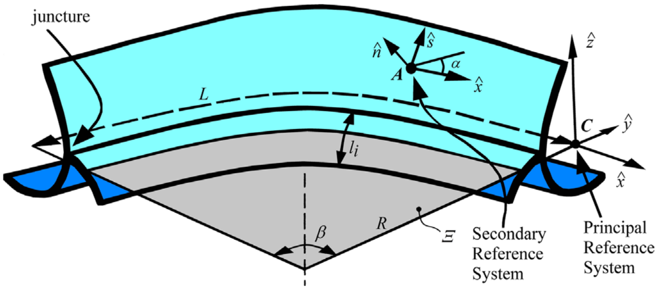

Fig. 1. Sketch of the thin-walled curved beam with the reference systems.