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Imperfection sensitivity analysis of laminated folded plates

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ABSTRACT

A novel methodology for imperfection sensitivity analysis is presented. Koiter's perturbation method is used to calculate the imperfection paths emanating from mode interaction bifurcations, which occur on the post-buckling paths of the single modes. The Monte Carlo method is used to test a large number of modes and all possible interactions among them. The computational cost is low because of the efficiency of Koiter's method. The demands of Koiter's method for accurate evaluations of higher order derivatives of the potential energy are met by a mixed, corotational element.

1. Introduction

The design of composite structures is most often dominated by buckling [1,2]. For example, the demands for fuel efficiency are prompting the aircraft industry to revolutionize airframe construction to save weight, and thus fuel. A promising concept is to let the airframe operate in the postbuckling regime, where the skin of the composite stiffened panels are allowed to buckle in normal flight conditions. This hinges upon the assumption that stiffened panels, and thus the entire airframe, are imperfection insensitive.

Imperfection sensitivity analysis requires the identification of a large number of buckling modes and their interaction. Because of the large number of possible modes and our a priori ignorance about which one would interact with each other, such analysis is prohibitively time consuming. Continuation methods based on Riks scheme are often used [3]. In spite of the simplicity of its numerical implementation, which requires only an approximation of the tangent stiffness matrix, the method suffers in the case of multiple bifurcations, requiring ad-hoc branch switch algorithms [4]. Continuation methods are time consuming, requiring a lengthy analysis for each assumed imperfection. Furthermore, type and shape of imperfections are unknown, either because the structure is in the design stage or because it is too difficult to measure them.

Therefore, the aim of this work is to propose a robust and efficient methodology to calculate the imperfection sensitivity of laminated composite folded plates, including stiffened panels as a particular case. The proposed methodology does not require a priori knowledge of the shape and magnitude of imperfections and does not rely on lengthy continuation analysis. Instead, it uses Koiter's perturbation approach [5,6] to calculate the bifurcation load, post-buckling path, and interaction between modes to detect bifurcations on the post-buckling path of individual modes, as well as the paths emanating from those bifurcations. The requirement for linearity of the constitutive equations is easily met by composite materials, which have a broad, linear stress and strain range of operation in compression [7].

The most recent implementations of Koiter's approach include spatial beam assemblages [8], folded plates [9], and composite structures [10]. Since the approach is based on fourth-order energy expansion [8], a finite element capable of accurately representing fourth-order terms is required for robustness of the analysis. The corotational approach [11,12] fulfils this requirement allowing the complete reuse of a linear element for geometrically nonlinear analysis. A mixed formulation is used to avoid extrapolation locking [13]. The recent 3D plate finite element [14] based on Hellinger–Reissner variational formulation guarantees an accurate evaluation of linear elastic response and of rotation fields [15], so it is very suitable to be used with a corotational formulation to obtain a geometrically nonlinear formulation, which is accurate up to fourth order energy terms [10, Fig. 3b].

Koiter's method provides robust prediction of the path emanating form interaction bifurcations between three or more modes, thus providing a good estimate of the imperfection sensitive, post-buckling trajectory (even when the shape and magnitude of the imperfections are unknown) that otherwise would be very costly to follow by a continuation methods. Mode interaction

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often produces the most deleterious imperfection sensitive path with the larger drop in load carrying capacity [16–18]. The difficulty resides on how to select the set of modes that produces the worst behavior.

The Monte Carlo method is proposed herein to find the modes that yield the most unfavorable, imperfection sensitive path. Although Monte Carlo is an expensive method, the computational cost is keep low thanks to the efficiency of both the element used and Koiter's approach. Also, Koiter's approach is quite demanding about the quality of higher order (up to 4th order) derivatives of the energy, but the element formulation used in this work is uniquely suited to satisfy those demands for accuracy at a low computational cost. The proposed methodology allows us to run thousand of analysis in a few seconds, obtaining the worst imperfection using a Monte Carlo simulation.

2. Koiter's formulation

2.1. The asymptotic analysis

Asymptotic analysis is essentially the implementation of Koiter's nonlinear elastic stability approach [5] into the finite element method (FEM) [6]. The solution process is based on an expansion of the potential energy Φ in terms of load factor λ and modal amplitudes ξ_i. It can be summarized as follows:

(i) The *fundamental path* is obtained as a linear extrapolation

$$\boldsymbol{u}^f[\lambda] = \boldsymbol{u}_0 + \lambda \hat{\boldsymbol{u}} \tag{1a}$$

where $\boldsymbol{u}_0$ is an initial displacement, possibly null, and $\boldsymbol{u} = \lambda\hat{\boldsymbol{u}}$ is the vector of kinematic parameters, i.e., the space of *degrees of freedom (dof)* of the structure, and $\hat{\boldsymbol{u}} = d\boldsymbol{u}/d\lambda$ is obtained as the solution of the linear algebraic equation

$$\boldsymbol{K}_0 \hat{\boldsymbol{u}} = \hat{\boldsymbol{p}} \tag{1b}$$

where $\hat{\boldsymbol{p}}$ is the reference load and $\boldsymbol{K}_0 = \boldsymbol{K}[\boldsymbol{u}_0]$ is the stiffness matrix, which contains the coefficients of the quadratic terms of the energy Φ''.

(ii) A cluster of *buckling loads* λ_i, $i = 1\ldots m$, and associated *buckling modes* $\dot{\boldsymbol{v}}_i$ are obtained along $\boldsymbol{u}^f[\lambda]$ by the critical condition

$$\boldsymbol{K}[\lambda_i]\dot{\boldsymbol{v}}_i = \boldsymbol{0}, \quad \boldsymbol{K}[\lambda] = \boldsymbol{K}[\boldsymbol{u}_0 + \lambda\ \hat{\boldsymbol{u}}] \tag{1c}$$

the eigenvalue problem is defined as fully nonlinear, to correctly recover the post-critical behavior. The nonlinearity is introduced by updating the configuration along the fundamental path.

Note that the size m of the subspace of buckling modes needed for the analysis is orders of magnitude smaller than the number of dof used to discretize the structure, often as little as $m=3$.

We denote by $\mathcal{V} = \{\dot{\boldsymbol{v}} = \sum_{i=1}^{m} \xi_i \dot{\boldsymbol{v}}_i\}$ the subspace spanned by the buckling modes $\dot{\boldsymbol{v}}_i$ (where ξ_i are the modal amplitudes) and by $\mathcal{W} = \{\boldsymbol{w} : \boldsymbol{w} \perp \dot{\boldsymbol{v}}_i, i = 1\cdots m\}$ its orthogonal complement, defined by the orthogonality condition

$$\boldsymbol{w} \perp \dot{\boldsymbol{v}}_i \Leftrightarrow \Phi_b''' \hat{u}\dot{v}_i w = 0 \tag{1d}$$

where $\hat{u} = \mathcal{L}\hat{\boldsymbol{u}}$, $\dot{v}_i = \mathcal{L}\dot{\boldsymbol{v}}_i$, $w = \mathcal{L}\boldsymbol{w}$ and $\mathcal{L}$ is the linear operator of FEM interpolation.

We denote by λ_b an appropriate reference value for the cluster, e.g., the smallest of λ_i or their mean value. Accordingly, a suffix "b" denotes quantities evaluated in correspondence to $\boldsymbol{u}_b = \boldsymbol{u}^f[\lambda_b]$.

(iii) Defining $\xi_0 = (\lambda - \lambda_b)$ and $\dot{\boldsymbol{v}}_0 = \hat{\boldsymbol{u}}$, the asymptotic approximation for any equilibrium path is approximated by a expansion in terms of mode amplitudes ξ_k as follows:

$$\boldsymbol{u}[\lambda, \xi_k] = \boldsymbol{u}_b + \sum_{i=0}^{m} \xi_i \dot{\boldsymbol{v}}_i + \frac{1}{2}\sum_{i,j=0}^{m} \xi_i \xi_j \boldsymbol{w}_{ij} \tag{1e}$$

where $\boldsymbol{w}_{ij} \in \mathcal{W}$ are quadratic corrections introduced to satisfy the projection of the equilibrium equation (see[19, Section 3.3]) into $\mathcal{W}$, obtained by the linear *orthogonal equations*

$$\delta\boldsymbol{w}^T(\boldsymbol{K}_b \boldsymbol{w}_{ij} + \boldsymbol{p}_{ij}) = 0, \quad \forall \boldsymbol{w} \in \mathcal{W} \tag{1f}$$

where $\boldsymbol{K}_b = \boldsymbol{K}[\boldsymbol{u}^f[\lambda_b]]$ and vectors $\boldsymbol{p}_{ij}$ are defined as a function of modes $\dot{\boldsymbol{v}}_i$; $i = 0\ldots m$, by the energy equivalence $\delta\boldsymbol{w}^T\boldsymbol{p}_{ij} = \Phi_b''' \delta w \dot{v}_j \dot{v}_j$.

(iv) The following energy terms are computed for $i,j = 0\ldots m$, $k = 1\ldots m$:

$$\begin{aligned}
&\mathcal{A}_{ijk} = \Phi_b''' \dot{v}_i \dot{v}_j \dot{v}_k \\
&\mathcal{B}_{ijhk} = \Phi_b'''' \dot{v}_i \dot{v}_j \dot{v}_h \dot{v}_k - \Phi_b''(w_{ij}w_{hk} + w_{ih}w_{jk} + w_{ik}w_{jh}) \\
&\mathcal{C}_{ik} = \Phi_b'' w_{00} w_{ik} \\
&\mu_k[\lambda] = \tfrac{1}{2}\lambda_b(\lambda - \tfrac{1}{2}\lambda_b)\Phi_b''' \hat{u}^2 \dot{v}_k + \tfrac{1}{6}\lambda_b^2(\lambda_b - 3\lambda)\Phi_b'''' \hat{u}^3 \dot{v}_k
\end{aligned} \tag{1g}$$

where the *implicit imperfection factors* μ_k are defined by the 4th order expansion of the unbalanced work on the fundamental path, i.e., $\mu_k[\lambda] = (\lambda\hat{p} - \Phi'[\lambda\hat{u}])\dot{v}_k$ (see [19, Eqs.(31,32)]).

(v) The equilibrium path is obtained by projecting the equilibrium equation [19, Section 3.4] on $\mathcal{V}$. According to Eqs. (1)–(1g), we have

$$\begin{aligned}
&\frac{1}{2}\sum_{i,j=0}^{m} \xi_i \xi_j \mathcal{A}_{ijk} + \frac{1}{6}\sum_{i,j,h=0}^{m} \xi_i \xi_j \xi_h \mathcal{B}_{ijhk} + \mu_k[\lambda] \\
&\quad - \lambda_b\left(\lambda - \frac{1}{2}\lambda_b\right)\sum_{i=0}^{m} \xi_i \mathcal{C}_{ik} = 0, \quad k = 1\ldots m
\end{aligned} \tag{1h}$$

Eq. (1h) is an algebraic nonlinear system of m equations in the $m+1$ variables ξ_0, $\xi_1\ldots\xi_m$, with known coefficients.

The software implementation of the asymptotic approach is quite easy and its computational cost remains of the order of that required by a standard linearized stability analysis [6]. Once the preprocessor phase of the analysis has been performed (Steps i–iv), the presence of small load and geometrical imperfections can be taken into account in the post-processing phase (Step v), by adding some, easily computed, additional imperfection terms in the expression of $\mu_k[\lambda]$, with a negligible computational cost, allowing for an inexpensive imperfection sensitivity analysis. From Eq. (1h) we can also extract information about the worst imperfection shapes [20,21] that we can use to improve the imperfection sensitivity analysis or for driving more detailed investigations through specialized path-following analysis [22].

2.2. Imperfection sensitivity analysis

The geometry and loads of thin-walled structures are affected by random distribution of small *imperfections*. In the proposed asymptotic method, the presence of small imperfections expressed by a load $\tilde{p}[\lambda]$ and/or an initial displacement $\tilde{u}$ affect Eq. (1g) only on the imperfection term $\mu_k[\lambda]$ that becomes [6]

$$\mu_k[\lambda] = \tfrac{1}{2}\lambda_b(\lambda - \tfrac{1}{2}\lambda_b)\Phi_b''' \hat{u}^2 \dot{v}_k + \tfrac{1}{6}\lambda_b^2(\lambda_b - 3\lambda)\Phi_b'''' \hat{u}^3 \dot{v}_k + \mu_k^g[\lambda] + \mu_k^l[\lambda] \tag{2}$$

with

$$\mu_k^g[\lambda] + \mu_k^l[\lambda] = \lambda\left(\Phi_b'' \hat{u}\tilde{u}\dot{v}_k - \tilde{p}[\lambda]\dot{v}_k\right) = \lambda\overline{\mu}_k \tag{3}$$

The aim of the *imperfection sensitivity analysis* is to link the presence of geometrical and load imperfections to the reduction of the limit load. For structures presenting coupled buckling modes, even a small load or geometrical imperfection may result in a