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Effect of in-plane boundary conditions on elastic buckling behavior of solid and perforated plates

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ABSTRACT

In this work, finite element method is used to obtain elastic buckling loads and mode shapes of plates. The effect of four different in-plane boundary conditions on the elastic buckling load of simply supported plates subjected to in-plane uni-axial compressive loading is studied. Elastic buckling behavior of plates with cut-outs (circular and square with curved corners) is also studied to illustrate the effect of the size, shape and the eccentricity of the cut-out on buckling loads. Results of the study show that the in-plane boundary conditions affect the elastic buckling behavior of the plate to a significant extent. Aspect ratio influences both the mode shape and the elastic buckling load for perforated plates, whereas its influence is generally limited to mode shape only in case of a solid plate. Restraints on the in-plane movements of plate's unloaded edges have different influence on the elastic buckling behavior of the plate when the position of cut-out is changed along the *x*-axis (loading direction) and along the *y*-axis (normal to loading direction) of the plate. Also, results show that a large cut-out in the vicinity of edges makes the plate unstable at a much lower load compared to the plate with no cut-out. It is envisioned that the results of this study may provide insight to predict buckling loads in practical scenarios.

1. Introduction

Thin structural plates are widely used as load resisting components in civil, mechanical and aerospace structures. Buckling of such plates is a common pattern of failure. Among various parameters that govern the buckling phenomenon of a thin plate, boundary conditions play an important (often critical) role in determining the buckling load [1,2]. Although the buckling phenomenon of plates has been studied for various out-of-plane boundary conditions and various in-plane loading cases, both theoretically and experimentally, it is clear from the available literature that a limited number of these studies have dealt with the effect of in-plane boundary conditions on the buckling behavior.

The effect of full and partial in-plane normal restraints, along the unloaded edges, on the buckling of plates was considered by Bedair and Sherbourne [3,4] and Bedair [5] for a plate loaded in-plane in two opposite edges in compression, bending and shear. It was concluded that the consideration of normal in-plane restraints along the unloaded edges results in a lower buckling load and a different mode shape than the plate under free in-plane edge movement. Effect of pre-buckling in-plane deformations on elastic buckling load was considered by Wang et al. [6] and Ziegler [7].

The effects of plate perforations and aspect ratio on its buckling were important areas of research during many past years. Roberts and Azizian [8] developed finite element-based solutions for buckling of plates having square and circular cut-outs and subjected to uniaxial and biaxial compressions, and pure shear. An approximate method was suggested by Narayanan and Chow [9] to predict the ultimate load and post-buckling behavior of perforated plates under uniaxial compressive loading. Yettram and Brown [10] proposed a direct matrix method for buckling analysis of square perforated plates having cut-outs of different sizes. The effect of size and position of circular cut-outs in simply supported and clamped plates subjected to uniaxial and biaxial compressive loadings and pure shear was studied by Sabir and Chow [11]. Study on mechanical and thermal buckling behavior of simply supported and clamped rectangular plates with circular and square cut-outs was carried out by Ko [12] using finite element approach. Ko considered in-plane normal restraints on the unloaded edges for both simply supported and clamped flexure boundary conditions. El-Sawy and Nazmy [13] studied the effect of aspect ratio on elastic buckling of simply supported plates with eccentric cut-outs. It was concluded that the buckling behavior in terms of buckling load and mode shapes of perforated rectangular plates may not necessarily be the same as that of the perforated square plates. The recommendation of that study was to provide a cut-out in predetermined location to avoid a drastic reduction in the buckling load. Minoru and Masahiko [14] conducted a finite element study

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to obtain the buckling and ultimate strength of simply supported rectangular plates with cut-outs subjected to longitudinal and transverse thrusts, and shear load. Simplified formulae were proposed based on that study. El-Sawy et al. [15] considered the elasto-plastic buckling behavior of simply supported perforated plates with different size of cut-outs to develop several design charts for elastic and elasto-plastic buckling stress versus slenderness ratio (ratio of width to the thickness of the plate) for plates with various slenderness ratios. Maiorana et al. [16] also considered the effect of position of circular and rectangular cut-outs on the elastic stability of simply supported rectangular plates subjected to in-plane compression and bending moment to propose the best orientation of a cut-out in a plate. Studies have also been conducted on elastic buckling and ultimate capacity of simply supported plates with eccentric circular and rectangular cut-outs [17,18]. Tajdari et al. [19] used finite element methods to study the effect of in-plane support conditions, aspect ratio and hole size on the buckling behavior of plates. Experimental investigations dealing with the buckling of plates have also been reported in the literature [20–24]. Many of these studies dealt with experimental determination of buckling and ultimate load capacity of simply supported and clamped rectangular plates with and without cut-outs.

In most of the aforementioned studies, some or all of the plate boundaries were assumed to be free for in-plane movements and thus, only out-of-plane boundary conditions were considered to predict the buckling behavior of the plate. Only few studies considered the effect of in-plane boundary restraints on the buckling phenomenon and were limited to the in-plane normal restraints on the unloaded edges of the plate. However, in practice, a plate element may have different in-plane restraints along the plate boundaries depending on its connections with the adjacent structural elements.

This study focuses on the effect of different in-plane boundary conditions on the buckling behavior of square and rectangular plates simply supported in out-of-plane direction, and uni-axially loaded in long direction with and without cut-out. At first, the effect of in-plane boundary conditions on the pre-buckling stress pattern in a solid plate under a uniformly applied compression is studied. The buckling behavior of the plate is then studied in terms of the buckling load and the corresponding mode shape. For plates with cut-out, the effect of size, shape and eccentricity of cut-out is also considered. Fig. 1 shows generalized schematic diagram of a plate with an eccentric cut-out showing the size and eccentricity of cut-out in x and y-directions.

2. Buckling with in-plane restraints

The elastic buckling load of a solid plate with length a and width b can be obtained by satisfying the equilibrium in a slightly bent configuration under the action of in-plane forces and then solving the associated eigenvalue problem. The equilibrium equation for a differential element of a plate under the action of

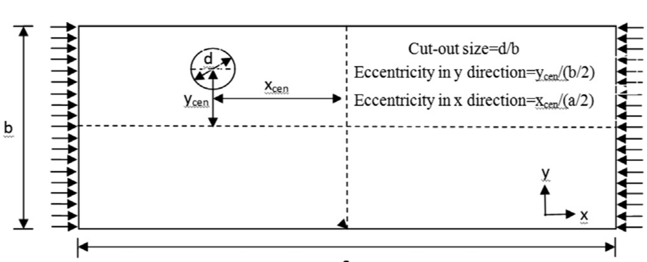

Fig. 1. Schematic diagram for generalized problem definition.

in-plane normal stresses σ_x, σ_y (along the long and short directions, respectively), and shear stress τ_{xy} is given by

$$D\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)=h\left(\sigma_x\frac{\partial^2 w}{\partial x^2}+2\tau_{xy}\frac{\partial^2 w}{\partial x \partial y}+\sigma_y\frac{\partial^2 w}{\partial y^2}\right) \quad (1)$$

where w is the out-of-plane displacement at the plate's mid-surface, h is the plate thickness, D is the flexural rigidity of plate given by $Eh^3/12(1-\nu^2)$, E is Young's modulus, and ν is Poisson's ratio. Eq. (1) with boundary conditions in terms of w can be expressed as an eigenvalue problem, whose solution yields the following expression for the critical stress:

$$\sigma_{cr}=\frac{kD\pi^2}{b^2 h} \quad (2)$$

In Eq. (2), k is a non-dimensional parameter, known as the buckling load factor or the buckling coefficient, which depends on the aspect ratio (i.e., a/b ratio) and the boundary conditions of the plate for a particular case of loading. When there are no in-plane restraints at the plate's edges, the stresses σ_x, σ_y, and τ_{xy} at any point are simply equal to the applied in-plane stresses. On the other hand, in-plane restraints on plate boundaries, which are often encountered in practical applications, cause a complex pre-buckling stress pattern and buckling mode. The analytical approach for such buckling problem with in-plane restraints requires the solution of the plane elasticity problem first to determine the actual in-plane stress pattern (σ_x, σ_y, and τ_{xy}) that is then used in solving Eq. (1) to get the critical stress applied to the plate. Owing to the complexity of the resulting equation for the in-plane elasticity problem, the finite element method has been used to obtain the buckling behavior of plate with different in-plane boundary conditions.

3. Numerical study

In the numerical investigation, plates are modeled in Abaqus [25], commercial finite element software, with elastic material having Young's modulus $E=210\,000$ N/mm^2, Poisson's ratio $\nu=0.3$ and a uniform thickness of 1.2 mm. Four different cases of in-plane boundary conditions or restraints are considered in this study while keeping the same out-of-plane boundary conditions. Table 1 provides details of these cases along with graphical representations, where u and v represent the mid-surface in-plane displacements in the loading direction and transverse to the loading direction, respectively. The effect of these in-plane restraints on the pre-buckling stress pattern of the plate is described in Section 3.4.

3.1. Mesh details

Plates are modeled using 8-node doubly curved thin shell element (designated as S8R5 in Abaqus), which uses 5 degrees-of-freedom (three translations and two in-plane rotations) and reduced integration. The element S8R5 is adopted based on its better capabilities when compared to other elements available in Abaqus [25] to model shell structures. Results for different elements sizes are generated by modeling a square simply supported plate of length 200 mm with no in-plane restraints (Case I) and subjected to in-plane uniaxial compressive forces (with same number of elements in both directions of the plane of the plate). Fig. 2 shows a comparison between the different shell elements to study their convergence rate to the well-known buckling load factor obtained from classical plate buckling theory. The comparison shown in Fig. 2 justifies the adoption of element S8R5 for this investigation, because of its computational efficiency and accuracy relative to other elements. A quite refined mesh with element