Thin-Walled Structures 90 (2015) 182–190

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Forced vibration analysis of arbitrarily constrained rectangular plates and stiffened panels using the assumed mode method

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A R T I C L E I N F O

Article history:
Received 25 November 2014
Received in revised form
22 January 2015
Accepted 22 January 2015
Available online 11 February 2015

Keywords:
Forced vibration
Mindlin plate
Stiffened panel
Point excitation
Boundary excitation
Assumed mode method

A B S T R A C T

This paper deals with numerical procedure for the vibration analysis of rectangular plates and stiffened panels subjected to point excitation force and enforced displacement at boundaries. The procedure is based on the assumed mode method, where natural response is determined by solving an eigenvalue problem of a multi-degree-of-freedom system matrix equation derived by using Lagrange's equation of motion. Mode superposition method is applied to calculate plate/stiffened panel frequency response. The Mindlin plate theory is adopted for a plate, while the effect of stiffeners having the properties of Timoshenko beams is taken into account by adding their potential and kinetic energies to the corresponding plate energies. The accuracy of the proposed procedure is justified by several numerical examples which include forced vibration analysis of plates and stiffened panels with different dimensions and framing sizes and orientations, having various combinations of boundary conditions. The results obtained by the developed in-house code are compared to those obtained by the finite element method (FEM) and experimental results from the relevant literature. The presented procedure is confirmed to be highly accurate.

1. Introduction

Plates are primary structural members in all fields of engineering; aeronautical, civil, mechanical, naval, ocean, etc. Moreover, stiffening of the plate is often used to increase its loading capacity and to prevent buckling [1]. At the same time, reinforcements significantly affect the dynamic properties of such structures and make the analysis rather complex. The most important engineering problems encountered with plate structures can be classified into three main groups: bending, stability and vibration [2].

In plate theory, two mathematical models are distinguished, the well-known Kirchhoff thin plate and the Mindlin thick plate theory [3]. In the former, shear influence on deflection and rotary inertia are small and are therefore ignored. There are different analytical and numerical methods for plate vibration analysis, both associated with some advantages and drawbacks, as elaborated in details in [4,5]. Analytical methods are quite simple to use, but can be applied only to plates with certain combination of boundary conditions. Different variants of the Rayleigh–Ritz method can be applied to plates with arbitrary boundary conditions, and their accuracy is dependent on the chosen set of orthogonal functions for the assumed natural modes.

In case of stiffened panels there is extensive literature on the static and natural vibration analysis using the finite element method (FEM), the boundary element method (BEM) or their combination [1,6]. However, to the authors' knowledge, a rather limited number of references is dedicated to the forced vibration of stiffened panels. Generally, the most common methods for the free vibration analysis of stiffened panels are: the closed-bound solutions; the energy methods and other numerical methods [7]. Olson and Hazell [8] predicted and measured the first 24 eigenfrequencies of stiffened plates using FEM and real-time laser holography. Fox and Sigillito [9] applied the Rayleigh–Ritz method to evaluate the eigenfrequencies of stiffened panels. Laura and Gutiérrez [10] analysed fundamental frequencies of rectangular plates elastically restrained only against rotation, assuming moderately thick plate. Liew et al. [11] presented the formulation of Mindlin–Engesser model for vibration analysis of

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http://dx.doi.org/10.1016/j.tws.2015.01.020
0263-8231/

moderately thick stiffened plates, based on the Ritz method. The boundary element method is applied to the aforementioned problem by Sapountzakis and Katsikadelis [12]. Further, Zeng and Bert [13] presented vibration analysis of stiffened plate by the differential quadrature method, separately treating the plate and stiffeners and their compatibility is modelled by their governing differential equations. An analytical-numerical method for quick natural vibration analysis suitable for thin rectangular plates reinforced by a small number of light stiffeners is presented by Dozio and Ricciardi [14]. Hongan et al. [15] have presented an analytical method for the vibration analysis of stiffened plates with arbitrary lengths and placement angles and examined the influence of different boundary conditions, coupling conditions, and reinforcing arrangements on dynamic response. Recently, Bedair [16] analysed free vibration characteristics of thin-walled plates with Zee-stiffeners using energy approach. During the last decades, FEM has become the most powerful tool in practical engineering. Therefore, it is not surprising that there are many references dealing with its application in free and forced vibration assessment of plate structures as well. Different issues related to the application of FEM in vibration of stiffened panels are considered in [7,17–19]. Also, Patel et al. [20] studied the effect of boundary conditions and stiffeners on the natural response of thin rectangular plates using FEM. Hamedani et al. [21] performed vibration analysis of stiffened panels using both conventional and super finite element method, and provided comparison with experimental results.

In spite of the fact that the finite element method is probably the most advanced tool for practical application, it may be rather time-consuming both in model generation and calculation execution. In this sense, in the early design stage when different topologies of plate structures are considered, it is preferable to use some simplified method. Therefore, a procedure based on the assumed mode method using characteristic orthogonal polynomials with the properties of Timoshenko beam functions is developed for the vibration analysis of plate structures. Up to now, it is applied only to the natural vibration analysis of plates [22], plates with openings [23], stiffened panels [6] and plate structures in contact with fluid [24].

Although the natural vibration analysis provides an insight in the structure dynamic behaviour and most of ordinary vibration problems are resolved by changing the system stiffness and moving the structure out of resonance, it is often necessary, especially in ships and offshore structures, to check compliance of vibration amplitudes with the prescribed criteria. However, vibration amplitudes are dependent on the excitation properties, and it is, therefore, necessary to perform forced vibration analysis. In this study, the developed procedure [6] is extended to the frequency response analysis of plates and stiffened panels subjected to point excitation force and enforced displacement at boundaries. Natural response is obtained by the assumed mode method, and then the mode superposition method is applied to calculate forced response. The method is validated with several numerical examples considering bare plates and stiffened panels having different properties and boundary conditions. Comparisons with FE and experimental results show that the presented procedure is highly accurate and reliable, despite its simplicity.

2. Equations of motion, formulation of system potential and kinetic energy

The Mindlin thick plate theory and Timoshenko beam theory are adopted for the plate and stiffeners, respectively [3,6]. Equations of motion of an isotropic rectangular Mindlin plate are derived from the equilibrium of sectional and inertia forces. Referring to the coordinate system in Fig. 1, equations of motion yield:

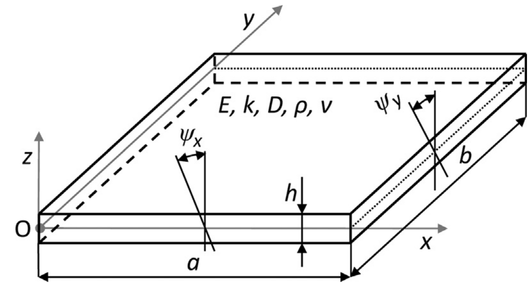

Fig. 1. Coordinates and schematic of rectangular Mindlin plate.

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_x}{\partial t^2}-D\left(\frac{\partial^2\psi_x}{\partial x^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_x}{\partial y^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_y}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial x}-\psi_x\right)=0, \tag{1}$$

$$\frac{\rho h^3}{12}\frac{\partial^2\psi_y}{\partial t^2}-D\left(\frac{\partial^2\psi_y}{\partial y^2}+\frac{1}{2}(1-\nu)\frac{\partial^2\psi_y}{\partial x^2}+\frac{1}{2}(1+\nu)\frac{\partial^2\psi_x}{\partial x\partial y}\right)-kGh\left(\frac{\partial w}{\partial y}-\psi_y\right)=0, \tag{2}$$

$$\frac{\rho}{kG}\frac{\partial^2 w}{\partial t^2}-\frac{\partial^2 w}{\partial x^2}-\frac{\partial^2 w}{\partial y^2}+\frac{\partial\psi_x}{\partial x}+\frac{\partial\psi_y}{\partial y}=0, \tag{3}$$

where ρ represents plate density, h is plate thickness, k is shear coefficient, t denotes time, while ν is Poisson's ratio. Further, D represents plate flexural rigidity $D=Eh^3/(12(1-\nu^2))$, while E and $G=E/(2(1+\nu))$ are Young's modulus and shear modulus, respectively.

Lagrange's equation of motion is used to formulate an eigenvalue problem:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)-\frac{\partial T}{\partial q_i}+\frac{\partial V}{\partial q_i}=0, \tag{4}$$

where V and T represent potential and kinetic energy of the considered system, respectively, and q_i is generalized coordinate.

Effect of stiffeners is taken into account by adding their potential and kinetic energies to the corresponding energies of a bare plate. In this sense, one can write:

$$V=V_p+V_s, \tag{5}$$

$$T=T_p+T_s, \tag{6}$$

where V_p and V_s are potential energies and T_p and T_s are kinetic energies of plate and stiffeners, respectively.

By introducing the non-dimensional parameters $\xi=x/a$, $\eta=y/b$, $\alpha=a/b$ and $S=kGh/D$, the following expressions are valid for the potential and kinetic energy of plate with length a and width b, having arbitrary edge constraints [22]:

$$\begin{aligned}V_p=\frac{D}{2\alpha}\int_0^1\int_0^1\Bigg[&\left(\frac{\partial\psi_\xi}{\partial\xi}\right)^2+\alpha^2\left(\frac{\partial\psi_\eta}{\partial\eta}\right)^2\\&+2\nu\alpha\frac{\partial\psi_\xi}{\partial\xi}\frac{\partial\psi_\eta}{\partial\eta}+\frac{1-\nu}{2}\left(\alpha\frac{\partial\psi_\xi}{\partial\eta}+\frac{\partial\psi_\eta}{\partial\xi}\right)^2\\&+S\left(\left(\frac{\partial w}{\partial\xi}-a\psi_\xi\right)^2+\alpha^2\left(\frac{\partial w}{\partial\eta}-b\psi_\eta\right)^2\right)\Bigg]d\xi d\eta\\&+\int_0^1\left[K_{Rx1}\psi_\xi^2(0,\eta)+SK_{Tx1}w^2(0,\eta)\right]d\eta\\&+\alpha^2\int_0^1\left[K_{Ry1}\psi_\eta^2(\xi,0)+SK_{Ty1}w^2(\xi,0)\right]d\xi\\&+\int_0^1\left[K_{Rx2}\psi_\xi^2(1,\eta)+SK_{Tx2}w^2(1,\eta)\right]d\eta\\&+\alpha^2\int_0^1\left[K_{Ry2}\psi_\eta^2(\xi,1)+SK_{Ty2}w^2(\xi,1)\right]d\xi,\end{aligned} \tag{7}$$