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A physically non-linear GBT-based finite element for steel and steel-concrete beams including shear lag effects

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ABSTRACT

This paper introduces an accurate and computationally efficient GBT-based finite element, specifically tailored to capture the materially non-linear behaviour of wide-flange steel and steel-concrete composite beams up to collapse. The element incorporates reinforced concrete cracking/crushing, shear lag effects and steel beam plasticity (including shear deformation of the steel web). A set of numerical examples is presented, showing that the proposed element is capable of capturing all relevant phenomena with a very small computational cost. In addition, analytical solutions for elastic shear lag are derived and the GBT modal decomposition features are employed to extract valuable information concerning the effect of shear lag phenomena up to collapse. For validation and comparison purposes, results obtained with shell/brick finite element models are also presented.

1. Introduction

The Generalised Beam Theory (GBT) is a thin-walled prismatic bar theory that extends Vlasov's classical beam theory [1] to effectively and efficiently handle cross-section in-plane and out-of-plane (warping) deformation, through the inclusion of so-called "cross-section deformation modes". GBT was introduced by Schardt almost 50 years ago [2,3] and it has since been continuously and considerably developed. Presently, GBT is well established as a very efficient and valuable tool for analysing prismatic thin-walled beams—see, e.g., [4–7].

In the field of steel-concrete composite bridge linear analyses without cracking effects, very promising results have been recently obtained with GBT, due to its straightforward capability of including shear lag and shear connection flexibility effects [8]. In particular, the effectiveness of the GBT approach was demonstrated for:

(i) linear elastic analyses, including cross-section distortion, the effect of transverse diaphragms, shear lag in wide flanges and shear connection flexibility;
(ii) vibration analyses (i.e., natural frequencies and vibration mode shapes), allowing for cross-section in-plane deformation and warping;
(iii) buckling analyses (i.e., lateral-distortional bifurcation loads and buckling mode shapes) of I-beams under constant hogging bending.

In all cases, it was shown that GBT can offer significant advantages with respect to the standard shell finite element and finite strip methods, namely due to its unique modal features, i.e., the fact that the kinematic description of the beam is based on a superposition of cross-section deformation modes with well-defined structural meanings, i.e., that represent specific effects, such as warping, local-plate bending, and distortion. This makes it possible to obtain accurate solutions with a small number of modes (and thus a small number of DOFs) and acquire valuable insight into the mechanics of the problem addressed through the inspection of the relative participations of the modes.

More recently, elastoplastic GBT formulations have also been developed for bifurcation [9], geometrically linear [10] or fully non-linear [11] analyses. These papers highlight that significant computational savings may be achieved, without sacrificing accuracy, if appropriate stress/strain components are set to zero and/or a shell-like stress resultant approach is employed. Further elastoplastic GBT applications are presented in [12].

This paper extends previous formulations by developing an efficient physically non-linear GBT-based beam finite element specifically aimed at capturing the global behaviour, up to collapse, of wide flange steel and steel-concrete composite beams. In particular, reinforced concrete non-linear behaviour is introduced and combined

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with shear lag effects and steel beam plasticity. It should be noted that modelling physically non-linear shear lag with beam elements is rather challenging, particularly for very wide flanges, since (i) the span-to-flange width ratio is small (which is somewhat in contradiction with the definition of a beam structural element) and, most importantly, (ii) the neutral surface is invariably located in the flange and the stresses vary rapidly across its thickness.

The formulation presented in this paper is characterised by an appropriate trade-off between accuracy and computational efficiency, which is achieved by taking advantage of the inherent characteristics of GBT and aiming at simplicity (a key goal), namely:

(i) Membrane shear deformation is only allowed in relevant cross-section zones, namely in wide flanges (to capture shear lag effects) and in the web of the steel girder (to capture vertical shear effects). These deformation types are allowed for through the inclusion of appropriate deformation modes.
(ii) The stress and strain fields are appropriately constrained in order to limit the number of admissible deformation modes and also to make it possible to employ simple material models for concrete and steel.

The outline of the paper is as follows. Section 2 starts by reviewing the fundamental aspects of a general physically non-linear GBT-based beam finite element. Then, the formulation is particularised for an efficient allowance of non-linear shear lag effects in wide flange steel and steel-concrete beams. Section 3 presents a set of applications, including numerical illustrative examples, concerning steel and steel-concrete beams, ranging from purely elastic problems (including analytical results) to more complex physically non-linear cases. For comparison and validation purposes, results obtained with shell and brick finite element models, using ADINA [13] and ATENA [14], respectively, are also presented. Finally, the paper closes with a few concluding remarks.

2. Finite element formulation

2.1. Fundamental equations

The beam finite element fundamental equations correspond to those employed in [10,11], although geometric non-linearity is not considered in the present work. With the Kirchhoff assumption for thin plates and the wall mid-surface local axes shown in Fig. 1, the displacement vector for each wall, $\boldsymbol{U}$, is expressed as

$$\boldsymbol{U}(x,y,z)=\begin{bmatrix}U_x\\U_y\\U_z\end{bmatrix}=\left(\overline{\Xi}_U(y)+z\Xi_U(y)\right)\begin{bmatrix}\boldsymbol{\phi}(x)\\\boldsymbol{\phi}_{,x}(x),\end{bmatrix}, \tag{1}$$

where the comma indicates a differentiation, $\boldsymbol{\phi}(x)$ is a column vector containing the deformation mode amplitude functions (the problem unknowns) and matrices $\overline{\Xi}_U(y), \Xi_U(y)$ read

$$\overline{\Xi}_U(y)=\begin{bmatrix}\mathbf{0}&\overline{\boldsymbol{u}}^t\\\overline{\boldsymbol{v}}^t&\mathbf{0}\\\overline{\boldsymbol{w}}^t&\mathbf{0}\end{bmatrix},\quad \Xi_U(y)=-\begin{bmatrix}\mathbf{0}&\overline{\boldsymbol{w}}^t\\\overline{\boldsymbol{w}}^t_{,y}&\mathbf{0}\\\mathbf{0}&\mathbf{0}\end{bmatrix}, \tag{2}$$

where $\overline{\boldsymbol{u}}(y)$, $\overline{\boldsymbol{v}}(y)$, $\overline{\boldsymbol{w}}(y)$ are column vectors containing the displacement functions of the wall mid-line along x, y, z, respectively, for each deformation mode. For arbitrary flat-walled cross-sections, these displacement functions may be determined from the procedures outlined in [15,16].

The non-null small-strain components are grouped in vector $\boldsymbol{\varepsilon}^t=[\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{xy}]$, which may be subdivided into membrane (M) and bending (B) components, viz.

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^M+\boldsymbol{\varepsilon}^B=\underbrace{\left(\Xi^M_\varepsilon(y)+z\Xi^B_\varepsilon(y)\right)}_{\Xi_\varepsilon(y)}\begin{bmatrix}\boldsymbol{\phi}\\\boldsymbol{\phi}_{,x}\\\boldsymbol{\phi}_{,xx}\end{bmatrix}, \tag{3}$$

with the auxiliary modal matrices

$$\Xi^M_\varepsilon(y)=\begin{bmatrix}\mathbf{0}&\mathbf{0}&\overline{\boldsymbol{u}}^t\\\overline{\boldsymbol{v}}^t_{,y}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&(\overline{\boldsymbol{u}}_{,y}+\overline{\boldsymbol{v}})^t&\mathbf{0}\end{bmatrix}, \tag{4}$$

$$\Xi^B_\varepsilon(y)=-\begin{bmatrix}\mathbf{0}&\mathbf{0}&\overline{\boldsymbol{w}}^t\\\overline{\boldsymbol{w}}^t_{,yy}&\mathbf{0}&\mathbf{0}\\\mathbf{0}&2\overline{\boldsymbol{w}}^t_{,y}&\mathbf{0}\end{bmatrix}. \tag{5}$$

In general, a plane stress state is assumed in each wall (varying along x, y, z) and the non-null components are grouped in vector $\boldsymbol{\sigma}^t=[\sigma_{xx}\ \sigma_{yy}\ \sigma_{xy}]$, which is related to the strains for a particular constitutive law . Within a Newton–Raphson iterative solution, the tangent stress–strain relation is written as

$$d\boldsymbol{\sigma}=\boldsymbol{C}_t\ d\boldsymbol{\varepsilon}, \tag{6}$$

where $\boldsymbol{C}_t$ is the consistent tangent constitutive matrix pertaining to the particular stress return mapping algorithm employed.

The finite element is obtained by interpolating the amplitude functions according to $\boldsymbol{\phi}=\boldsymbol{\Psi}\boldsymbol{d}$, where matrix $\boldsymbol{\Psi}$ contains the interpolation functions and vector $\boldsymbol{d}$ contains the unknowns (nodal values of the amplitude functions). Both Hermite (cubic) and Lagrange quadratic polynomials are employed, with the latter associated with the deformation modes that only involve warping displacements.

Finally, the out-of-balance force vector $\boldsymbol{g}$, the tangent stiffness matrix $\boldsymbol{K}_t$ and the incremental load vector $\Delta\boldsymbol{f}$ are obtained from the numeric integration of the expressions

$$\boldsymbol{g}=\int_V\begin{bmatrix}\boldsymbol{\Psi}\\\boldsymbol{\Psi}_{,x}\\\boldsymbol{\Psi}_{,xx}\end{bmatrix}^t\Xi^t_\varepsilon\ \boldsymbol{\sigma}\ dV-\int_\Omega\begin{bmatrix}\boldsymbol{\Psi}\\\boldsymbol{\Psi}_{,x}\end{bmatrix}^t\overline{\Xi}^t_U\ \overline{\boldsymbol{q}}\ d\Omega, \tag{7}$$

$$\boldsymbol{K}_t=\int_V\begin{bmatrix}\boldsymbol{\Psi}\\\boldsymbol{\Psi}_{,x}\\\boldsymbol{\Psi}_{,xx}\end{bmatrix}^t\Xi^t_\varepsilon\ \boldsymbol{C}_t\ \Xi_\varepsilon\begin{bmatrix}\boldsymbol{\Psi}\\\boldsymbol{\Psi}_{,x}\\\boldsymbol{\Psi}_{,xx}\end{bmatrix}dV, \tag{8}$$

$$\Delta\boldsymbol{f}=\int_\Omega\begin{bmatrix}\boldsymbol{\Psi}\\\boldsymbol{\Psi}_{,x}\end{bmatrix}^t\overline{\Xi}^t_U\ \Delta\overline{\boldsymbol{q}}\ d\Omega, \tag{9}$$

where $\overline{\boldsymbol{q}}$ are forces acting along the walls mid-surface Ω (for simplicity, volume forces are not considered).

2.2. Particularisation for shear lag analysis

Although a suitable physically non-linear GBT-based beam finite element is obtained with the formulas of the preceding section, no significant reduction of DOFs with respect to a standard shell model is achieved without further enhancements. Therefore, additional modifications and simplifications need to be introduced, by taking advantage of the particular characteristics of the problem being addressed: capture the global non-linear behaviour of steel and steel-concrete composite beams, including concrete/steel material non-linearity and shear lag. The particular modifications/simplifications adopted are described next.

It is assumed that the cross-sections are of the type shown in Fig. 1(b), comprising a concrete slab with longitudinal steel reinforcement and a steel I-beam. The corresponding wall mid-lines, which constitute the basis of the GBT kinematic description, are