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Review

Review: Constrained finite strip method developments and applications in cold-formed steel design

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ABSTRACT

The stability of thin-walled members is decidedly complex. The recently developed constrained Finite Strip Method (*c*FSM) provides a means to simplify thin-walled member stability solutions through its ability to identify and decompose mechanically meaningful stability behavior, notably the formal separation of local, distortional, and global deformation modes. The objective of this paper is to provide a review of the most recent developments in *c*FSM. This review includes: fundamental advances in the development of *c*FSM; applications of *c*FSM in design and optimization; identifying buckling modes and collapse mechanisms in shell finite element models; and, additional stability research initiated by the *c*FSM methodology. A brief summary of the *c*FSM method, in its entirety, is provided to explain the method and highlight areas where research remains active in the fundamental development. The application of *c*FSM to cold-formed steel member design and optimization is highlighted as the method has the potential to automate generalized strength prediction of thin-walled cold-formed steel members. Extensions of *c*FSM to shell finite element models is also highlighted, as this provides one path to bring the useful identification features of *c*FSM to general purpose finite element models. A number of alternative methods, including initial works on a constrained finite element method, initiated by *c*FSM methods, are also detailed as they provide insights on potential future work in this area. Research continues on fundamentals such as methods for generalizing *c*FSM to arbitrary cross-sections, improved design and optimization methods, and new ideas in the context of shell finite element method applications.

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1. Introduction

Thin-walled cold-formed steel members enjoy a relatively complicated stability response for typical geometries and loading. As a result, specialized tools for studying this stability response have been developed and advanced. One of the most successful of these tools has been the Finite Strip Method (FSM). In particular, the signature curve for member stability analysis popularized by Hancock [1] has provided the organizing thrust of today's member design: global, distortional, and local(-plate) buckling based on the signature curve.

In recent years, an additional tool: Generalized Beam Theory (GBT) has shown that the buckling deformations may be formally treated in a modal nature that mechanically separates global, distortional, local, and other modes [2]. This formal separation is integral to GBT, and allows measurement of modal participation. By extracting the mechanical assumptions that lead to the separation one may extend the definitions to other methods. In particular, this insight lead to the development of the constrained FSM (*c*FSM), which imbues FSM with the same ability as GBT, in terms of the separation of the deformations. In fact, the methods have been compared and shown to be nearly coincident in their end result [3–5].

This paper is a modified and significantly extended version of the paper presented at the CIMS2012 conference [6]. The paper provides a review of fundamental developments in *c*FSM as well as research results that are closely related and/or made possible by *c*FSM. This review focuses on the last three years, though older results are referenced and briefly presented if germane to understanding the latest results.

The paper begins, in Section 2, with a summary of the constrained finite strip method (*c*FSM). The method is built-up from the simplest case (simply supported ends) then extended to general end boundary conditions. Ongoing research in the basic assumptions and the definition of the modes is also summarized. Section 3 of the paper provides a summary of efforts to apply *c*FSM in a variety of design, optimization, and modal identification problems. The design efforts focus on the use of *c*FSM to automate the identification of modes for use in cold-formed steel member design. This process is further generalized in the examination of shape optimization of cold-formed steel members. The last topic in Section 3 focuses on the use of cFSM base functions for modal identification in shell finite element method (FEM) models. Specifically local, distortional, and global classifications are provided for elastic buckling, geometrically nonlinear, and full nonlinear collapse analysis of shell FEM models. Finally, in Section 4 a series of research results are discussed that are not directly linked to, but unquestionably initiated by, the constraining technique of *c*FSM, including nascent efforts in the constrained finite element method (*c*FEM).

2. The constrained finite strip method

2.1. Classic FSM

The finite strip method leverages the longitudinal regularity of many thin-walled members to dramatically decrease the problem size. Members are discretized into longitudinal strips per Fig. 1. Within a strip, local displacement fields u, v, and w are discretized as follows:

$$u = \sum_{m=1}^{q} \begin{bmatrix} \left(1-\frac{x}{b}\right) & \frac{x}{b} \end{bmatrix} \begin{Bmatrix} u_{1[m]} \\ u_{2[m]} \end{Bmatrix} Y_{[m]},$$

$$v = \sum_{m=1}^{q} \begin{bmatrix} \left(1-\frac{x}{b}\right) & \frac{x}{b} \end{bmatrix} \begin{Bmatrix} v_{1[m]} \\ v_{2[m]} \end{Bmatrix} Y'_{[m]} \frac{a}{\mu_{[m]}} \tag{1}$$

$$w = \sum_{m=1}^{q} \begin{bmatrix} 1-\frac{3x^2}{b^2}+\frac{2x^3}{b^3} & x\left(1-\frac{2x}{b}+\frac{x^2}{b^2}\right) & \frac{3x^2}{b^2}-\frac{2x^3}{b^3} & x\left(\frac{x^2}{b^2}-\frac{x}{b}\right) \end{bmatrix} \begin{Bmatrix} w_{1[m]} \\ \theta_{1[m]} \\ w_{2[m]} \\ \theta_{2[m]} \end{Bmatrix} Y_{[m]} \tag{2}$$

where the longitudinal shape function is

$$Y_{[m]} = sin(m\pi y/a) \tag{3}$$

the strip degrees of freedom (DOF): $u_{i[m]}$, $v_{i[m]}$, $w_{i[m]}$, $\theta_{i[m]}$ are indicated for the first term (m=1) of the simply supported (SS) end boundary condition in Fig. 1. Unlike FEM DOF, FSM DOF always occur at the same

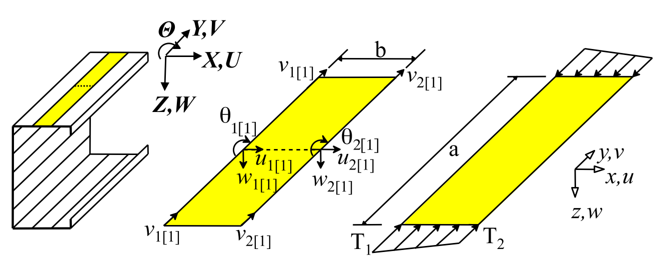

Fig. 1. Finite strip discretization, strip DOF, and notation.