Thin-Walled Structures 81 (2014) 101–107

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Local-overall interaction buckling of inelastic columns: A numerical study of the inelastic Van der Neut column

Jurgen Becque *

The University of Sheffield, Sheffield, South Yorkshire, UK

ARTICLE INFO

Available online 15 August 2013

Keywords:
Interaction buckling
Coupled instabilities
Inelastic behaviour

ABSTRACT

The elastic Van der Neut column has been a very useful tool to enhance our understanding of local-flexural interaction buckling of columns. In particular, Van der Neut's work has (a) illuminated the basic mechanism behind interaction buckling and (b) demonstrated that local-flexural interaction buckling is a highly imperfection sensitive phenomenon. In this paper, a component of inelastic behaviour is added to the classical Van der Neut column and its influence on the column buckling curve and the imperfection sensitivity is investigated. Numerical calculations are carried out using a finite differences scheme which is first explained in detail. In general, plasticity gives rise to a second 'plateau' in the buckling curve, curtailing the column capacity at low column slenderness values. This plateau typically exhibits low to moderate imperfection sensitivity. However, for yield stresses of the order of the local buckling stress of the flanges, both plateaus merge to form an extended zone of very high imperfection sensitivity.

1. Introduction

In 1969 Van der Neut published a paper which became one of the most influential and seminal pieces of work in the field of coupled instabilities. The paper provided a theoretical framework for the interaction mechanism of local and overall flexural buckling of columns. In order to simplify the problem and draw clear, straightforward conclusions from the work, Van der Neut considered an idealized column (now known as the 'Van der Neut column') which consisted of two flanges, simply supported along the longitudinal edges by infinitely thin webs (Fig. 1). The function of the webs is purely to provide support to the flanges, however they do not contribute in carrying any axial load. Van der Neut's column consisted of a purely elastic material and no consideration was given to plasticity.

The importance of Van der Neut's work mainly lies in two important achievements. First of all, using the idealized column Van der Neut was able to clearly explain the mechanics of the interaction of local and overall buckling and uncover the reasons behind the signature shape of the curve plotting the buckling load vs. the length (or, in more general terms, the slenderness) of the column. Central to the understanding is the fact that local buckling results in a sudden reduction of the axial compressive stiffness of the flanges. Van der Neut thereby borrowed from the work by Hemp [6], who demonstrated that the axial post-buckling stiffness of a compressed plate is nearly constant over an extended range of displacements and equal to $\eta = 0.41$ times the initial (pre-buckling) stiffness. Consequently, the Euler curve shifts down by a factor η above the local buckling load. For a perfect column without imperfections the column curve displays a plateau at the level of the local buckling load, which can be divided into a stable and an unstable region, depending on whether the post-buckling capacity of the column calculated on the basis of Engesser's [4] double modulus exceeds the local buckling load or not. In experiments on actual columns with imperfections the theoretical plateau is often observed as a flattening of the column curve around the local buckling load (e.g. [2,3]).

Second, apart from considering a column with perfectly straight flanges Van der Neut [12] studied the effect of adding either a local or an overall imperfection. He conclusively demonstrated that local-overall interaction buckling is a phenomenon which is highly imperfection sensitive, and especially so when the local and overall buckling stresses are of the same magnitude. Local and overall imperfections thereby have similar effects on the column buckling curve. In this respect it should be noted that in a second paper on the topic Van der Neut [13] made some amendments to his earlier calculations pertaining to the influence of an overall imperfection, which however did not alter the main conclusions of his original paper nor detract in any way from its importance.

It is also noted that the conclusions drawn for the Van der Neut column are in some sense an illustration of a more general and well-known principle which links the lack of post-buckling capacity (in this case in the region $L_0 < L < L_1$) to a heightened imperfection sensitivity in the same region.

* Tel.: +44 114 2220252.
E-mail addresses: jurgen_becque@hotmail.com, J.Becque@Sheffield.ac.uk

0263-8231/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.tws.2013.07.010

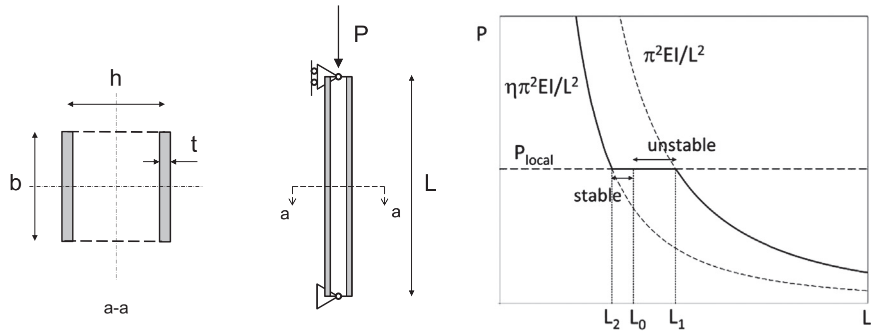

Fig. 1. The Van der Neut column.

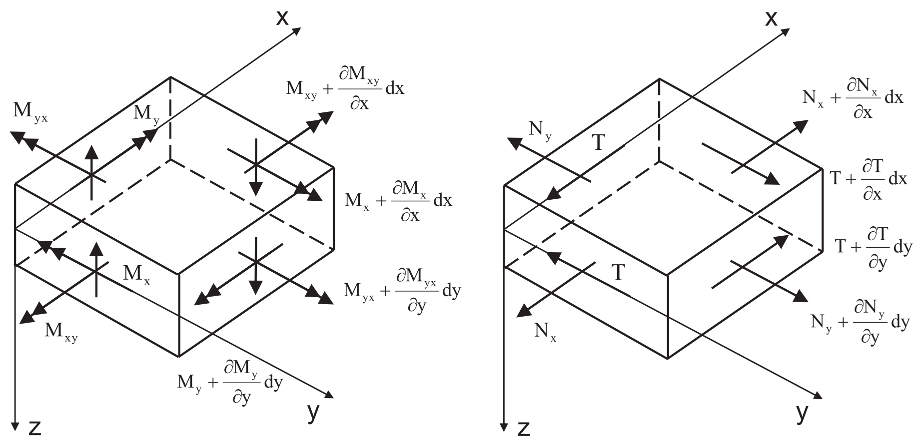

Fig. 2. Stress resultants on an infinitesimal plate element.

Van der Neut′s work only considered a column made of a purely elastic material. The aim of this paper is to add a component of inelastic behaviour to the Van der Neut column and study its effect on (a) the overall shape of the column buckling curve and (b) the imperfection sensitivity of the problem.

2. Post-buckling behaviour of a single plate

2.1. Strain–displacement relations

In order to arrive at an understanding of the buckling behaviour of our idealized column, we must first devote some attention to the post-local buckling behaviour of a single inelastic plate. With the addition of plasticity it is obvious that the post-buckling stiffness can no longer be assumed constant and a more comprehensive model is needed.

Seen the fact that large plate deflections are typically encountered in the post-buckling range, strain–displacement relationships are employed which retain the second order terms in the plate deflections w, but neglect higher order terms in the (smaller) membrane displacements u and v:

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2-z\frac{\partial^2 w}{\partial x^2} \tag{1}$$

$$\varepsilon_y=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2-z\frac{\partial^2 w}{\partial y^2} \tag{2}$$

$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}-2z\frac{\partial^2 w}{\partial x\partial y} \tag{3}$$

The x- and y-axes are thereby located in the mid-plane of the plate with the z-axis pointing along the normal, as indicated in Fig. 2. Eqs. (1)–(3) imply that plane sections remain plane after bending, an assumption typically maintained into the inelastic range.

Since the correct modelling of plasticity inherently necessitates an incremental approach, Eqs. (1)–(3) are re-written in incremental form by differentiation with respect to time:

$$\dot{\varepsilon}_x=\frac{\partial \dot{u}}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial \dot{w}}{\partial x}-z\frac{\partial^2 \dot{w}}{\partial x^2} \tag{4}$$

$$\dot{\varepsilon}_y=\frac{\partial \dot{v}}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial \dot{w}}{\partial y}-z\frac{\partial^2 \dot{w}}{\partial y^2} \tag{5}$$

$$\dot{\gamma}_{xy}=\frac{\partial \dot{u}}{\partial y}+\frac{\partial \dot{v}}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial \dot{w}}{\partial y}+\frac{\partial \dot{w}}{\partial x}\frac{\partial w}{\partial y}-2z\frac{\partial^2 \dot{w}}{\partial x\partial y} \tag{6}$$

where $\dot{\varepsilon}_x$, $\dot{\varepsilon}_y$ and $\dot{\gamma}_{xy}$ are the normal strain and shear strain increments, $\dot{w}$ is the incremental plate deflection and w is the (total) plate deflection at any given point. $\dot{u}$ and $\dot{v}$are the incremental displacements in the x- and y-direction respectively. In Eqs. (4)–(6), as well as in the remainder of the paper, a 'dot' represents differentiation with respect to time.

2.2. Constitutive equations

For the purpose of modelling plasticity the Von Mises yield surface is adopted with an associated flow rule and isotropic hardening. The flow rule, equivalent to the Prandtl–Reuss equations,