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# Mode interaction in thin-walled equal-leg angle columns

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ABSTRACT

This paper presents and discusses numerical results, obtained through Ansys shell finite element analyses, dealing with the post-buckling behaviour (mostly elastic, but also elastic–plastic), ultimate strength and failure mode nature of fixed-ended and pin-ended thin-walled equal-leg angle steel columns with coincident critical flexural-torsional and minor-axis flexural buckling loads (*i.e.*, experiencing very strong coupling effects between these two global instability phenomena) – for comparative purposes, columns that are buckling in pure flexural-torsional and flexural modes are also analysed. Since the main aim of the work is to investigate the column imperfection-sensitivity, the analyses concern otherwise identical columns containing initial geometrical imperfections with different shapes and amplitudes, combining the competing critical buckling modes – particular attention is paid to the sign of the minor-axis flexural component. The results reported consist of column (i) elastic equilibrium paths and the corresponding peak loads and deformed configurations and (ii) elastic–plastic collapse loads and mechanisms, making it possible to assess how they are influenced by the initial geometrical imperfections.


## 1. Introduction

Thin-walled columns with all their walls sharing a common longitudinal edge (*e.g.*, angle, T-section and cruciform columns) exhibit no primary warping (if the minute rounded corner effects are neglected, of course) and thus, their warping resistance stems exclusively from secondary warping – this resistance has been found to be quite non-negligible in the presence of end support warping fixity [1–3]. Nevertheless, this feature implies a very low torsional stiffness, thus rendering the above columns highly susceptible to instability phenomena involving torsion (torsional or flexural-torsional buckling). Since this work deals exclusively with fixed-ended and pin-ended (singly symmetric) equal-leg angle columns with short-to-intermediate lengths, the involvement of torsion is felt through flexural-torsional buckling. At this stage, it should be pointed out the support conditions of the fixed-ended (F) and pin-ended (P) angle columns only differ in the minor-axis flexural rotations, which are fully restrained in the first case and completely free in the second case – particularly, it is worth noting that both these support conditions, which are commonly considered in column tests, fully prevent the end section warping displacements, torsional rotations and major-axis flexural rotations, which implies that F and P columns exhibit exactly the same major-axis flexural and torsional behaviour (stiffness and strength).

The structural behaviour of fixed-ended and pin-ended equal-leg angle columns has been investigated, experimentally and/or numerically, by several authors. Experimental campaigns were reported (i) by Popovic et al. [4], Young [5] and Mesacasa Jr. [6], for F columns, and (ii) by Wilhoite et al. [7], Popovic et al. [4], Chodraui [8] and Maia et al. [9], for P columns. The fairly large experimental failure load data obtained by the above researchers, as well as the considerable number of numerical ultimate loads determined by Ellobody and Young [10], Camotim et al. [11], Silvestre et al. [12] and Dinis et al. [13], were used to develop and validate several design approaches for equal-leg angle columns – those proposed by Young [5] (F columns), Rasmussen [14,15] (P columns) and Silvestre et al. [12] (F and P columns) deserve to be specially mentioned.[1]

At this stage, it is worth mentioning that the post-buckling behaviour and ultimate strength of short-to-intermediate equal-leg angle columns is governed by the interaction between two global buckling modes, (i) one combining major-axis flexure and torsion corresponding to the almost horizontal plateau of the $P_{cr}$ *vs.* $L$ curve shown in Fig. 1(a) ($P_{cr}$ and $L$ are the column critical buckling load and length) and (ii) the other involving exclusively

[1] Very recently, Dinis et al. [13] claimed to have found more rational design approaches for F and P columns, which involve genuine flexural-torsional buckling concepts (instead of the “traditional” local buckling ones).

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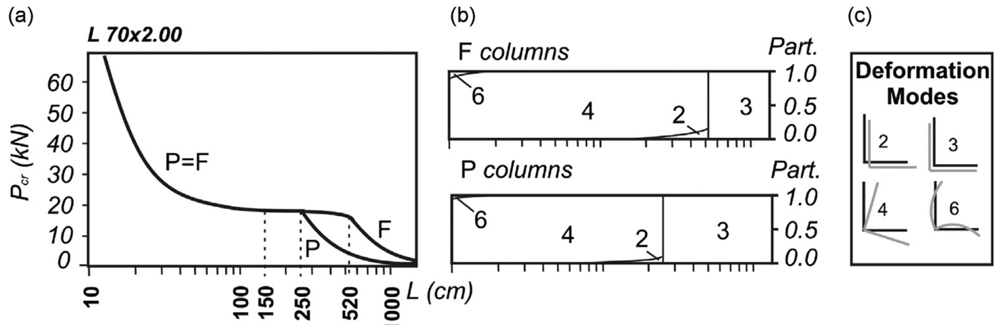


**Fig. 1.** Buckling behaviour of F and P equal-leg angle columns: (a) $P_{cr}$ *vs. L* curves, (b) GBT modal participation diagrams, and (c) in-plane shapes of the GBT deformation modes **2**-**6**.

minor-axis flexure corresponding to the descending branch of the aforementioned $P_{cr}$ *vs. L* curve [3]. The GBT (Generalised Beam Theory) modal participation diagrams displayed in Fig. 1(b) make it possible to quantify the relative contributions of the various deformation modes, depicted in Fig. 1(c), to the column critical buckling modes [1]. Moreover, the observation of the results presented in Figs. 1(a)–(c), which concern F and P angle steel ($E$=210 GPa and $\nu$=0.3) columns with cross-section dimensions $70\times70\times2.0$ mm (rounded corner effects disregarded), show that (i) the short-to-intermediate and intermediate-to-long columns buckle in flexural-torsional (**2**+**4**)[2] and pure flexural (**3**) modes, respectively and (ii) the "transition length" separating the two buckling behaviours is naturally much lower for the P columns than for their F counterparts (basically due to the 75% minor-axis flexural buckling load drop).

As it would be logical to expect, it was also found [3] that the (detrimental) influence of the coupling between the two global competing buckling modes on the angle column post-buckling behaviour and ultimate strength increases considerably as the length approaches the respective "transition value" (coincident flexural-torsional and flexural critical buckling loads). Furthermore, this coupling is much stronger in the *P* columns, due to the absence of minor-axis end moments to counteract the load eccentricity effects stemming from the effective centroid shifts occurring in the advanced post-buckling stages (*e.g.*, [16]).[3]

Concerning the design of F and P angle columns, even if the most performing available methodologies, namely those developed by Young [5], Rasmussen [14,15] and Silvestre et al. [12], ultimate strength estimates that compare quite well with the available experimental and numerical data, it must be recognised that such procedures are based on local-global interactive buckling concepts (*i.e.*, disregard the fact that short-to-intermediate columns buckle in flexural-torsional modes) and have a predominant empirical nature. This implies that they do no seize the real column collapse mechanics, thus reflecting the current lack of in-depth knowledge about all the aspects involved in the angle column non-linear structural behaviour. Such knowledge is indispensable to search for more rational design procedures, combining accuracy and safety with a mechanical meaning as solid/realistic as possible. One key aspect requiring proper understanding is the interaction between flexural-torsional and flexural buckling, a topic seldom investigated in the past – indeed, the existing studies on thin-walled column mode interaction concern almost only global and local (or, more recently, distortional) buckling. Moreover, the investigation on this global interactive behaviour is further complicated by the angle (i) deceptive geometrical simplicity (just two outstands) and (ii) mono-symmetry (no symmetry with respect to the minor-axis), which greatly affects the column imperfection-sensitivity.

This work aims at contributing to overcome the shortcomings mentioned in the previous paragraph, by presenting and discussing numerical results concerning the (i) post-buckling behaviour (mostly elastic, but also elastic–plastic), (ii) ultimate strength and (iii) failure mode nature of F and P thin-walled equal-leg angle steel columns experiencing different levels of flexural-torsional/flexural (FT–F) interaction: (i) very strong interaction, which corresponds to columns with lengths associated with coincident FT and F critical buckling loads (*i.e.*, the "transition length" separating the "almost horizontal plateau" and the descending branch of the $P_{cr}$ *vs L* curve illustrated in Fig. 1(a)), and (ii) less strong interaction, which corresponds to lengths below (FT critical buckling) and above (F critical buckling) the "transition length" just defined.

The numerical results are obtained through Ansys [19] shell finite element analyses (SFEA) based on (i) column discretisations into fine four-node isoparametric element meshes (length-to-width ratio close to 1), (ii) a steel material behaviour that is either linear elastic or linear-elastic/perfectly plastic stress–strain curve (residual stresses and corner effects disregarded) and (iii) column end supports modelled by attaching rigid plates to the end cross-sections, thus ensuring full warping and local displacement/rotation restraints. The rigid end-plate are then either (i) clamped (F columns), which prevents all the global displacements/rotations (except the axial displacement, of course), or (ii) placed on cylindrical hinges that allow minor-axis flexural rotations (P columns) – *i.e.*, F and P columns only differ in the minor-axis flexural rotation restraint.

In order to investigate the column imperfection-sensitivity associated with the FT–F interaction, the results are obtained from analyses that concern sets of otherwise identical columns that contain initial geometrical imperfections with different shapes and/or amplitudes (various combinations of the normalised competing FT and F buckling mode shapes). Particular attention is paid to the sign of the minor-axis flexural component, which is expected to play a major role in influencing the column response. The results displayed consist of (i) elastic equilibrium paths and associated peak loads, (ii) deformed configurations and displacement profiles, and (iii) elastic–plastic collapse loads, which make it possible to assess how these equilibrium paths, deformations and loads are influenced by the initial geometrical imperfection shape and/or amplitude for the columns with the various lengths

[2] GBT modal participation diagrams in Fig. 1(b) clearly show that the column buckling modes (i) are predominantly torsional and (ii) exhibit a (small) major-axis flexural component that grows with the length.

[3] It is still worth mentioning that the experimental set-up commonly adopted to test pin-ended specimens involves "rigid links" connecting the pinned supports (cylindrical hinges) to the rigid plates attached to the column end cross-sections (to ensure warping-fixity). Mesacasa Jr. et al. [17] have recently found that these rigid links have the net effect of lowering the column minor-axis flexural buckling load, thus rendering it more susceptible to the coupling effects dealt with in this work and contributing to a more pronounced ultimate strength erosion.