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The behaviour of pin-ended flange elements in compression

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ABSTRACT

Stowell's solution [1] for the buckling behaviour of flange elements in compression was premised on the assumption that the element was fixed against flexural rotations at the ends, a condition representing relatively thick elements for which the thickness dimension is adequate to prevent rotations. This paper presents a solution similar to Stowell's which is applicable to pin-ended flange elements. Aspects not considered in Stowell's work, such as the use of elliptic functions to describe the gradual change of mode shape from sinusoidal to essentially linear, and the gradual and asymptotic changes in axial rigidity in the post-buckling range are described in the paper. The paper also presents comparisons between the behaviour of pin-ended and fixed-ended flange elements. Finally, simple strength equations for flange elements in uniform compression based on the first yield criterion are derived.

1. Introduction

Stowell [1] presented an analysis of the buckling behaviour of flange elements based on a solution to the equation for nonlinear torsion of a cruciform in compression. The analysis was premised on the assumption that the element was fixed against flexural rotations at the ends, a condition representing relatively thick elements for which the thickness dimension is adequate to prevent rotations, as shown in Fig. 1a. However, thin-walled metal construction today is increasingly concerned with very thin elements for which the resistance to rotation is small and overcome soon after buckling commences. Such elements are more appropriately modelled as pin-ended, as shown in Fig. 1b. This paper presents a solution to a pin-ended flange element based on Stowell's work.

2. Elliptic integrals and functions

There exist elliptic integrals of the first, second and third kinds, e.g. see [2]. The elliptic integrals of the first and second kinds are defined as,

$$u = F(\varphi, k) = \int_0^{\varphi} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2\alpha}} \tag{1}$$

and

$$E(\varphi, k) = \int_0^{\varphi} \sqrt{1 - k^2 \sin^2\alpha}\, d\alpha \tag{2}$$

respectively, where k is a parameter known as the elliptic modulus, $(0 \le k < 1)$, and φ is the amplitude of u, or $\varphi = \mathrm{am}(u)$. When $\varphi = \pi/2$, the integrals above are said to be "complete", and the following notation is used,

$$K(k) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2\alpha}} \tag{3}$$

and

$$\overline{E}(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\alpha}\, d\alpha \tag{4}$$

The elliptic sine (sn), cosine (cn) and delta (dn) functions are defined as inversions of u as follows:

$$\mathrm{sn}(u, k) = \sin\varphi \tag{5}$$

$$\mathrm{cn}(u, k) = \cos\varphi \tag{6}$$

$$\mathrm{dn}(u, k) = \sqrt{1 - k^2 \sin^2\varphi} \tag{7}$$

The (k)'s will be omitted in utilising the notation, for brevity (e.g. $\mathrm{sn}(u, k) \equiv sn(u)$). The elliptic sine function has the following behaviour at the limits of the elliptic modulus:

$$\mathrm{sn}(u, 0) = \sin u \tag{8}$$

and

$$\lim_{k \to 1}(\mathrm{sn}(u, k)) = \tanh u \tag{9}$$

This transition of the elliptic sine function from a circular sine to a hyperbolic tangent explains the change of shape of the solutions for the twist rotation and longitudinal strain as buckling progresses (Section 3).

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3. Solution for the twist rotation

Consider a cruciform column consisting of four identical flange elements of width b and thickness t, as shown in Fig. 2a. The element is subject to a uniform shortening (δ) under the action of the axial compression force (P).

Stowell's work is founded in the observation that the buckling displacement of flange elements is essentially linear across the width of the element, as shown in Fig. 2c. As such, the solution for the buckling of a flange element can be obtained as the solution for the torsional buckling of a cruciform in uniform compression.

The differential equation for the twist rotation (θ) of a member in non-linear non-uniform torsion [1,3] is,

$$\left(GJ - P\frac{I_p}{A}\right)\frac{d\theta}{dx} - EI_w\frac{d^3\theta}{dx^3} + \frac{1}{2}EI_n\left(\frac{d\theta}{dx}\right)^3 = T \tag{10}$$

where $T=0$ (for this problem), $G=E/(2(1+\nu))$ and the geometric properties are taken for a single flange element as follows: $A=bt$, $I_p=1/3b^3t$, $J=1/3bt^3$, $I_w=1/36b^3t^3$ and $I_n=4/45b^5t$. Substituting these expressions into Eq. (10) and using the notation $(\ )' \equiv d(\ )/dx$, the equilibrium equation is obtained as,

$$\theta''' + \frac{12}{t^2}\left[\varepsilon_{av} - \frac{(t/b)^2}{2(1+\nu)}\right]\theta' - \frac{8}{5}\left(\frac{b}{t}\right)^2(\theta')^3 = 0 \tag{11}$$

where the average strain is defined as

$$\varepsilon_{av} = \frac{P}{EA} \tag{12}$$

On making the substitutions,

$$\gamma_b = b\theta' \tag{13}$$

$$\xi = \frac{x}{t} \tag{14}$$

$$m^2 = 12\left(\varepsilon_{av} - \frac{(t/b)^2}{2(1+\nu)}\right) \tag{15}$$

where γ_b is the rotation of the fibre at the free edge, as shown in Fig. 2d, the governing equation is obtained as,

$$\ddot{\gamma}_b + m^2\gamma_b - \frac{8}{5}\gamma_b^3 = 0 \tag{16}$$

in which $\dot{(\ )} \equiv d(\ )/d\xi$. The solution to Eq. (16) is [1],

$$\xi + \xi_0 = \pm\int_0^{\gamma_b}\frac{d\gamma_b}{\left[c^2 - m^2\gamma_b^2 + 4/5\gamma_b^4\right]^{1/2}} \tag{17}$$

where ξ_0 and c are arbitrary integration constants to be obtained from the boundary conditions. It is here assumed that the twist rotation is periodic and antisymmetric about the end cross-section while symmetric about the centre, as shown in Fig. 1b, whereby the boundary conditions may be expressed as $\gamma_b=0$ at $x=0$ and $x=L$, where the origin of the x-axis is chosen at mid-length. The condition $\gamma_b=0$ at $x=0$ implies that $\xi_0=0$.

Making the substitutions,

$$g^2 = \frac{m^2}{2} + \sqrt{\frac{m^4}{4} - \frac{4}{5}c^2} \tag{18}$$

$$h^2 = \frac{m^2}{2} - \sqrt{\frac{m^4}{4} - \frac{4}{5}c^2} \tag{19}$$

and defining the variables,

$$\frac{1}{g}\sin\psi = \frac{\gamma_b}{c} \tag{20}$$

and

$$k = \frac{h}{g} \tag{21}$$

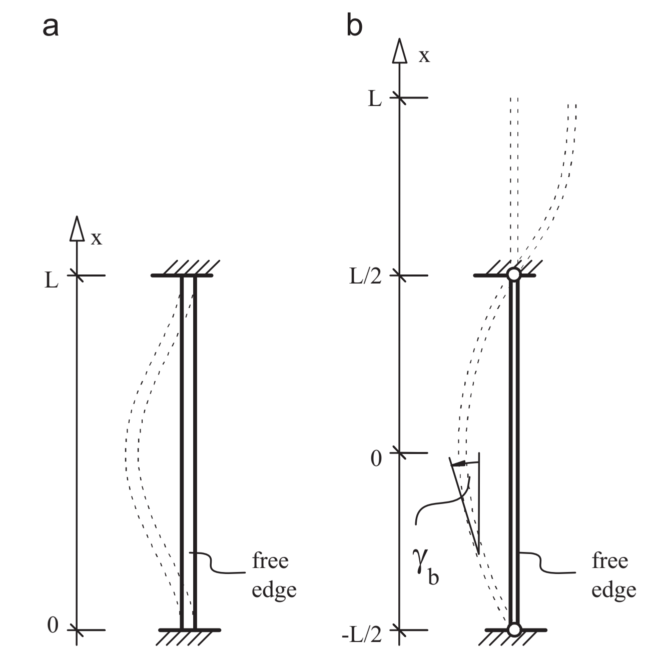

Fig. 1. Fixed and pinned end conditions. (a) Fixed-ended boundary conditions, (b) Pin-ended boundary conditions.

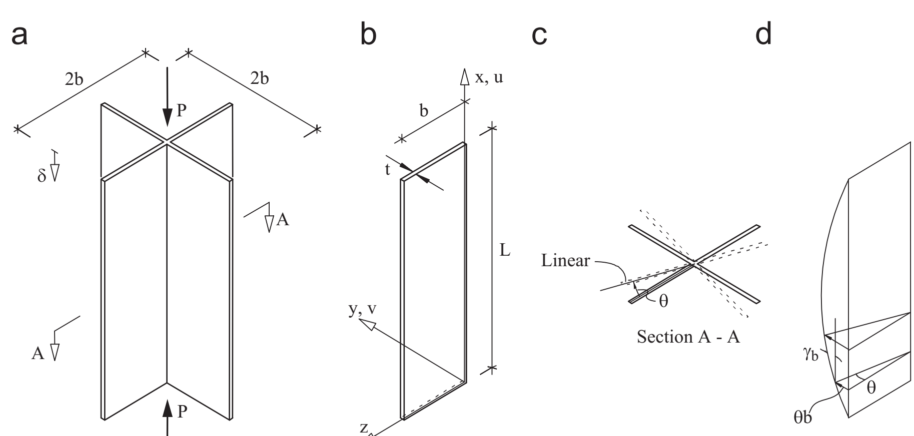

Fig. 2. (a) Cruciform in compression, (b) flange element, (c) torsional buckling, and (d) fibre rotation.