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Buckling of cracked functionally graded plates under tension

Ahmad Amiri Rad*, Danial Panahandeh-Shahraki

Department of Mechanical Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran

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ABSTRACT

Buckling of functionally graded cracked plates under tension has not been investigated so far. In this paper critical buckling load of functionally graded plates containing a crack has been obtained using classical plate theory through the finite element method. Displacement in vicinity of crack tips has been approximated using previous solutions related to bending of cracked plates. Effect on buckling of plate under uni-axial and bi-axial tension of different parameters, such as plate dimensions and material properties, are studied. Results show that the critical load decreases as material gradient index increases, while bi-axial loading leads to higher critical loads compared to uni-axial case.

1. Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composites that have a smooth and continuous variation of material properties with spatial coordinates. FGMs are highly resistant to temperature gradients and are preferred to laminate composites because they are not composed of different plies and are safe against delamination. The advantages of FGMs have increased their popularity in engineering applications and have attracted the attention of researchers to gain a better understanding of their mechanical behavior. Buckling is a mode of failure which a structure can experience in certain situations. Imperfections like cracks, which can be created in a FGM structure deliberately or undesirably during the production or use of the structure can affect the buckling behavior. Therefore the study of buckling of structures like plates which contain cracks can provide useful information to designers.

Many studies have been done on mechanical and thermal buckling of functionally graded plates (FGPs) with no cracks. Shariat et al. [1] presented the exact solution for buckling behavior of rectangular functionally graded plates with geometrical imperfections using classical plate theory. The plate was assumed to be under in-plane compressive loads. Shariat and Eslami [2] obtained the exact solution for buckling of rectangular thick functionally graded plates under mechanical and thermal loads, using the third-order shear deformation plate theory. Wu et al. [3] found analytical solution for the post-buckling response of the functionally graded plate, subjected to thermal and mechanical loads, based on the first-order shear deformation plate theory. Zhao et al. [4] investigated the mechanical and thermal buckling of functionally graded plates based on first-order shear deformation plate theory, using the element-free kp-Ritz method. Bodaghi and Saidi [5] solved the equilibrium and stability equations for buckling of thick functionally graded rectangular plates based on the higher-order shear deformation plate theory using Levy-type analytical solution. Latifi et al. [6] studied the effect of various boundary conditions, using Fourier series expansion, on the buckling of thin rectangular functionally graded plates subjected to proportional biaxial compressive loadings based on classical plate theory. Lal et al. [7] used the nonlinear finite element method for post-buckling analysis of functionally graded plates under mechanical and thermal loads using higher-order shear deformation theory.

Buckling of cracked plates has been studied by many researchers. Shaw and Huang [8] presented the finite element formulation for the buckling analysis of homogeneous cracked plates subjected to uniaxial tensile loads using on von Karman's theory. Kumar and Paik [9] extracted the governing differential equation for the buckling of homogeneous plates with edge and central crack under uniaxial and biaxial compressive and in-plane shear loads. They used the hierarchical trigonometric functions which satisfied various boundary conditions. Brighenti used the finite element method for the buckling analysis of variously cracked rectangular homogeneous thin-plates under tension and compression [10,11], and shear loads [12]. He studied the effect of variation of mechanical properties such as Poisson's ratio on critical buckling load. Seifi and Khoda-yari [13] carried out experimental and numerical studies on the critical

* Corresponding author.
E-mail addresses: ahmad.amiri2@gmail.com (A. Amiri Rad), d.panahandeh@me.iut.ac.ir (D. Panahandeh-Shahraki).

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buckling load of homogeneous plates with central inclined crack subjected to uni-axial compression. Pan et al. [14] introduced an improved hybrid semi-analytical method for calculating elastic buckling load of a thin homogeneous plate with a central straight through-thickness crack subjected to axial compression using Raleigh–Ritz energy method. Several works have been done on vibration of cracked functionally graded beams [15,16] and plates [17–21].

Buckling of cracked functionally graded beams has been studied by few researchers. Ke et al. [22] studied the post-buckling response of beams made of functionally graded materials containing an edge crack, based on Timoshenko beam theory and von Karman nonlinear kinematics using Ritz method. Yang and Chen [16] presented analytical solutions for the free vibration and the buckling of functionally graded beams with edge cracks by using Bernoulli–Euler beam theory and the rotational spring model.

To our knowledge only one work [23] has been done on buckling of functionally graded plates containing cracks. While the focus of this work is on buckling of cracked functionally graded plates under compression, buckling of cracked plates under tension is also crucial in design problems; in fact a cracked plate can also buckle under tension because of large compressive stresses created near cracked area.

In the current study the buckling behavior of functionally graded plates under uni-axial and bi-axial tension loads has been investigated. The classical plate theory (CLPT) considering von Karman's moderate rotation kinematics has been used in framework of finite elements to obtain critical buckling loads for different plate dimensions, crack orientations and mechanical properties. For elements surrounding the crack tips special formulation has been used which relies on previous solution of bending of cracked plates. Pre-buckling solution has been obtained prior to solving buckling eigenvalue problem, using quarter-point elements around crack tips.

2. Fundamental equations

Specifications and dimensions of studied plate are depicted in Fig. 1. The geometry includes length a, width b, total thickness h, and length of crack d. The crack has angle γ with respect to the y axis.

2.1. Kinematics

Based on classical plate theory, the displacement field is defined as follows [24]:

$$u(x,y,z) = -z\frac{\partial w_0}{\partial x}, \quad v(x,y,z) = -z\frac{\partial w_0}{\partial y}, \quad w(x,y,z) = w_0(x,y) \tag{1}$$

where u, v, and w are the Cartesian components of displacement field at any generic point (x,y,z) in the x, y, and z directions, respectively. w_0 shows lateral displacement of plate's mid-surface.

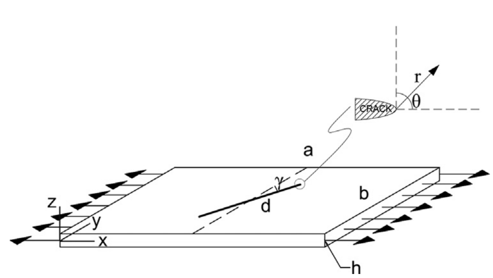

Fig. 1. Plate with central crack under uni-axial tension.

In the present study, the displacement in the in-plane directions on the mid-surface are assumed to be zero.

The von-Karman's strains can be written as follows [25]:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right) \tag{2}$$

The strain–displacement relation can be rewritten in the vector form, as follows:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{(\mathbf{NL})} + z\,\boldsymbol{\varepsilon}^{(1)}, \boldsymbol{\varepsilon} = [\,\varepsilon_{xx} \quad \varepsilon_{yy} \quad \gamma_{xy}\,] \tag{3}$$

where

$$\boldsymbol{\varepsilon}^{(\mathbf{NL})} = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}^{(1)} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{4}$$

2.2. Constitutive law for functionally graded materials

In a functionally graded material some properties of the material change continuously as a specific function of the coordinates. In present study, it is assumed that material only changes along thickness by Reddy's power law; for an arbitrary property it can be written as follows [24]:

$$p(z) = p_t V_t(z) + p_b(1 - V_t(z)) \tag{5}$$

where p_t and p_b are the property's values at the top and bottom of the plate. The term V_t is called volume fraction and can be expressed as:

$$V_t(z) = \left(\frac{2z+h}{2h}\right)^n \tag{6}$$

where n is called gradient index and z is the coordinate along thickness as shown in Fig. 1. For $n=0$ this equation represents a homogeneous plate. In the current study one of the most common types of FGM plates, which consists of pure ceramic at top and pure metal at bottom, is considered.

Using the thin plate theory for a functionally graded plate, the stress–strain relation, can be written as follows [25]:

$$\boldsymbol{\sigma} = \mathbf{Q}\boldsymbol{\varepsilon}, \quad \boldsymbol{\sigma}^{\mathbf{T}} = \{\,\sigma_{xx} \quad \sigma_{yy} \quad \tau_{xy}\,\}^{\mathbf{T}} \tag{7}$$

where $\mathbf{Q}$ is the reduced elastic stiffness matrix, defined as:

$$\mathbf{Q} = \frac{E(z)}{(1-\upsilon^2)}\begin{pmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{pmatrix} = \mathbf{A_1}E(z) \tag{8}$$

Therefore the moments created by the stresses associated to the linear part of $\boldsymbol{\varepsilon}$ (see Eq. (3)), $\mathbf{M} = \{\,M_x \quad M_y \quad M_{xy}\,\}^T$ could be obtained as [25]:

$$\mathbf{M} = \int_{-h/2}^{h/2} \boldsymbol{\sigma}\, z dz = \mathbf{D}\boldsymbol{\varepsilon}^{(1)} \tag{9}$$

where $\mathbf{D} = \mathbf{A_1}(\int_{-h/2}^{h/2} E(z)z^2 dz)$.

2.3. Finite element model

2.3.1. Stability equation

In this section, we extract the flexural and geometric stiffness matrices by applying variational principles to potential energy functions. The total potential energy of a single plate element can be divided in two parts [25]:

$$\Pi = \Pi_S + \Pi_{Ext} \tag{10}$$