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Post-local-buckling of fiber-reinforced plastic composite structural shapes using discrete plate analysis

Pizhong Qiao [a,b,*], Qingyuan Chen [a]

[a] State Key Laboratory of Ocean Engineering and School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China

[b] Department of Civil and Environmental Engineering, Washington State University, Sloan Hall 117, Pullman, WA 99164-2910, USA

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ABSTRACT

In this paper, post-buckling of rectangular composite plates rotationally restrained at the longitudinal unloaded edges and subjected to end shortening strain at the simply-supported loaded edges is analyzed using the first-order shear deformation plate theory-based spline finite strip method, and its application to post-local-buckling of fiber-reinforced plastic (FRP) composite structural shapes is illustrated with discrete plate analysis. Two cases of elastically- and rotationally-restrained plates are analyzed using the spline finite strip method: rotationally-restrained along both the unloaded boundary edges (RR) and one rotationally-restrained and the other free along the unloaded edges (RF). The two cases of rotationally-restrained plates (i.e., the RR and RF plates) are further treated as the discrete plates of closed and open section FRP shapes, and by considering the effect of elastic restraints at the joint connections of flanges and webs, post-local-buckling of various FRP shapes under end shortening is studied. The numerical comparisons with the finite element modeling demonstrate that the proposed discrete plate analysis technique and spline finite strip method can be used as an efficient and valid tool for post-local-buckling analysis of FRP shapes.

1. Introduction

The panels with either simply-supported or clamped supported on their unloaded edges are often ideal ones which are the limited boundary conditions in analysis. It is well known that the panels are often restrained at the boundaries in practice. To the authors' knowledge, there are only a few existing studies in the literature on the post-buckling analysis of panels with elastically-restrained boundary conditions. Rhodes and Harvey [1] presented an analysis for post-buckling behavior of thin flat plates in compression with their unloaded edges elastic restraint against rotation using energy method. Khong and Ong [2] performed a post-buckling analysis of a thin flat rectangular plate with unloaded edges subjected to various support boundary including equal or unequal degree of elastic restraint against rotation and translation based on the principle of minimum potential energy. Bisagni and Vescovini [3] proposed an analytical formulation for the study of local skin buckling load and nonlinear post-buckling behavior of isotropic and composite stiffened panels subjected to axial compression. In their study, the skin is modeled as a thin plate, while the stiffeners are considered as torsion bars in the analysis. Stamatelos et al. [4] developed an analytical methodology for local buckling and post-buckling behavior of isotropic and orthotropic stiffened plates. Their approach considered the stiffened panel segment located between two stiffeners, and the remaining panels are replaced by the equivalent transverse and rotational springs of varying stiffness and act as the elastic edge supports.

The finite strip method is an effective and versatile numerical tool, and it has been used for post-local-buckling analysis of prismatic structures by many authors. Graves-Smith and Sridharan [5,6] first investigated the post-local-buckling behavior of isotropic prismatic thin walled structures under end shortening using the finite strip method. Hancock et al. [7–9] studied the post-local-buckling of thin-walled structures applying both the finite strip method and spline finite strip method. Dawe et al. [10] described a finite strip method for analysis of the post-local-buckling behavior of composite laminated, orthotropic prismatic plate structures subjected to progressive uniform end shortening. Ovesy et al. [11] developed a semi-energy finite strip approach for the post-local-buckling analysis of geometrically-perfect thin-walled prismatic structures under uniform end shortening.

Fiber-reinforced plastic (FRP) composite structural shapes have been commonly used in the aerospace, automotive, marine and construction industries. Problems associated with large elastic

* Correspondence to: Department of Engineering Mechanics, Shanghai Jiao Tong University, Mulan Building A903, Shanghai 200240, PR China.
E-mail addresses: qiao@sjtu.edu.cn, qiao@wsu.edu (P. Qiao).

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deformation and local buckling/post-buckling are common in current design of FRP shapes [12,13]. The local buckling analysis of FRP shapes could be investigated by modeling the flanges and webs individually and considering the flexibility of the flange-web connection. In this type of simulation, each flat panel of prismatic FRP shapes is modeled as a composite plate subjected to elastic rotational restraints along the unloaded edges (i.e., the flange-web connection) [12,13]. The webs of box-, I-, C- and Z-sections can all be modeled as a plate rotationally- and elastically-restrained at two unloaded edges (RR) and loaded in compression at two opposite edges (Fig. 1(a)) [14]. Similarly, the flanges or webs of T- and L-sections and the flanges of I-, C- and Z-sections can all be simulated as a plate rotationally- and elastically-restrained at one unloaded edge and free at the other unloaded edge (RF) (see Fig. 1(b)) [15].

In this paper, the post-buckling behavior of FRP plates elastically- and rotationally-restrained along the unload edges and under the end shortening is presented using the spline finite strip method, and the accuracy of the method is validated against the results of finite element analysis. Then, the method is used to predict the post-local-buckling behaviors of pultruded FRP shapes, by considering the discrete laminated plates or panels as components (e.g., flanges or webs) of FRP shapes rotationally-restrained along one or both unloaded edges. The discrete plate analysis is validated by comparing the post-buckling behavior of the rotationally-restrained plates from the FRP shapes using the proposed spline finite strip method with those of the whole FRP shapes by both the spline finite strip and finite element methods.

2. Theoretical formulations

2.1. Cubic B-spline function

The cubic B-spline function with equal spacing is chosen as the longitudinal functions in the finite strip analysis. The uniform cubic B-spline function (Fig. 2) is expressed as

$$\varphi_i=\frac{1}{6\ h^3}\begin{cases} 0 & y<y_{i-2} \\ (y-y_{i-2})^3 & y_{i-2}\le y\le y_{i-1} \\ h^3+3h^2(y-y_{i-1})+3h(y-y_{i-1})^2-3(y-y_{i-1})^3 & y_{i-1}\le y\le y_i \\ h^3+3h^2(y_{i+1}-y)+3h(y_{i+1}-y)^2-3(y_{i+1}-y)^3 & y_i\le y\le y_{i+1} \\ (y_{i+2}-y)^3 & y_{i+1}\le y\le y_{i+2} \\ 0 & y>y_{i+2} \end{cases} \tag{1}$$

If the longitudinal direction is divided into M equal spaced sections, there are a total of $M+3$ spline interpolation parameters. The longitudinal displacement function $f(y)$ can be approximately assumed by a linear combination of base functions $\varphi_i(y)$ as

$$f(y)=\sum_{i=-1}^{M+1}\alpha_i\varphi_i(y)\ \text{ or }\ f(y)=\{\varphi\}^{\mathrm{T}}\{\alpha\} \tag{2}$$

where the base function vector and the interpolation parameter vector are $\{\varphi\}=[\varphi_{-1}\quad \varphi_0\quad \varphi_1\quad \dots\quad \varphi_{M-1}\quad \varphi_M\quad \varphi_{M+1}]^{\mathrm{T}}$ and $\{\alpha\}=[\alpha_{-1}\quad \alpha_0\quad \alpha_1\quad \dots\quad \alpha_{M-1}\quad \alpha_M\quad \alpha_{M+1}]^{\mathrm{T}}$, respectively.

In order to satisfy the prescribed boundary conditions, the variables α_{-1} and α_{M+1} can be replaced by the function values of $f(y)$ at $y=0$ and a, respectively. The modified vector $\{\beta\}$ can be obtained by

$$\{\beta\}=[T]\{\alpha\} \tag{3}$$

where $\{\beta\}=[f(0)\quad \alpha_0\quad \alpha_1\quad \dots\quad \alpha_{M-1}\quad \alpha_M\quad f(a)]^{\mathrm{T}}$, and

$$[T]=\begin{bmatrix} \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & & & & \\ 0 & 1 & 0 & & & & \\ 0 & 0 & 1 & & & & \\ & & & \ddots & & & \\ & & & & 1 & 0 & 0 \\ & & & & 0 & 1 & 0 \\ & & & & \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix}$$

Eq. (2) then becomes

$$f(y)=\{\overline{\varphi}\}^{\mathrm{T}}\{\beta\} \tag{4}$$

where $\{\overline{\varphi}\}=\{\varphi\}^{\mathrm{T}}[T]^{-1}$.

2.2. Spline finite strip method

Based on the first order shear deformation plate theory (FSDPT), the displacements of the middle surface of the plate $u'(x,y)$, $v'(x,y)$, $w'(x,y)$, and the rotations of the normal to the middle surface $\phi_x{}'(x,y)$ and $\phi_y{}'(x,y)$ under the end shortening strain ε can be written as

$$\begin{aligned} u'(x,y)&=u \\ v'(x,y)&=\varepsilon\left(\tfrac{a}{2}-y\right)+v \\ w'(x,y)&=w \\ \phi_x'(x,y)&=\phi_x \\ \phi_y'(x,y)&=\phi_y \end{aligned} \tag{5}$$

In the context of spline finite strip analysis, the displacements u, v and w, and the rotations ϕ_x and ϕ_y are expressed as a product of

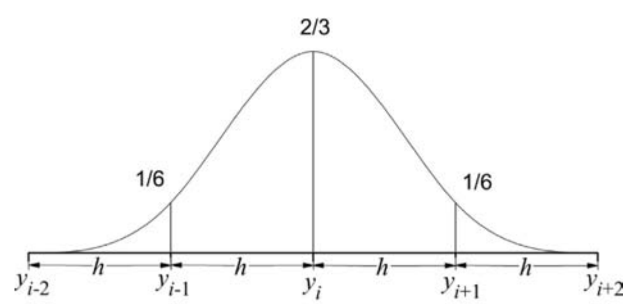

Fig. 2. The uniform local cubic B-spline function $\varphi_i\,(y)$

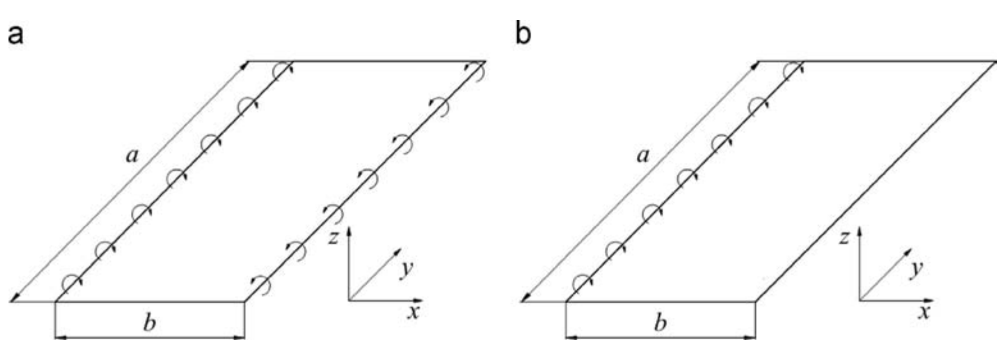

Fig. 1. Rotationally- and elastically-restraned plate elements in FRP shapes. (a) RR plate element (b) RF plate element.