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Generalized constrained finite strip method for thin-walled members with arbitrary cross-section: Primary modes

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ABSTRACT

In this paper the generalization of the constrained finite strip method (*c*FSM) is discussed. *c*FSM is a special version of the semi-analytical finite strip method (FSM), where carefully defined constraints are applied which enforce the thin-walled member to deform in accordance with specific mechanics, e.g., to allow buckling only in flexural, lateral–torsional, or a distortional mode. In the original *c*FSM only open cross-section members are handled, here the method is extended to cover any flat-walled member, including those with closed cross-sections or cross-sections with open and closed parts. Moreover, in the original *c*FSM only 4 deformation classes are defined, here the deformation field is decomposed into additional, mechanically meaningful, sub-fields. Formal mechanical criteria are given for the deformation classes, and implementation of the criteria regardless of cross-section topology is illustrated. In this paper, the primary deformation classes are presented in detail. Primary deformations are associated with minimal cross-section discretization, i.e. nodal lines located at folds and ends only. This paper is accompanied by a companion, where secondary modes and additional practical aspects in the selection of base vectors for the deformation classes are discussed. With the proposed modifications the powerful *c*FSM capabilities of buckling mode decomposition and identification are extended to essentially arbitrary thin-walled cross-sections.

1. Introduction

Thin-walled members possess complicated stability behavior. If subjected primarily to longitudinal stresses, three characteristic buckling classes are usually distinguished: global (G), distortional (D), and local-plate (L) buckling. Although in practical situations these modes rarely appear in isolation, typically some modal coupling is involved, the GDL classification has still been found useful for capacity prediction, and appears either implicitly or explicitly in current thin-walled design standards, e.g. [1,2]. Capacity prediction requires the critical loads associated with the various buckling classes. In case of the Direct Strength Method [1] the buckling mode classification is directly applied. However, critical load calculation for the various buckling classes is also part of the Effective Width Method [2], though in some cases the critical loads are embedded in the background of the provided formulae. Critical load calculation for thin-walled members is usually accomplished by one of three numerical methods: (i) the shell finite element method (FEM), (ii) generalized beam theory (GBT), and/or (iii) the finite strip method (FSM).

FEM, by using shell finite elements, is general and can be used to analyze almost any thin-walled member. However, FEM is not able to decompose the various buckling classes, which often makes capacity calculation ambiguous. As a result, shell FEM is essential in research, but not efficient in practical design.

GBT has shown that buckling deformations may be formally treated in a modal nature that mechanically separates global, distortional, local, and other deformations. This formal separation is integral to GBT, and allows pure buckling mode calculations and measurements of modal participation in coupled modes. GBT was first worked out for unbranched open and simple closed cross-sections [3], then later extended to more general flat-walled cross-sections [4] as well as to some curved cross-sectional shapes (e.g., [5]). A free-to-use computer program (GBTUL) has also been worked out, which first handled basic cross-section shapes only [6], then very recently extended to more general cross-sections [7].

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FSM is based on the work of Cheung [8], but popularized by Hancock [9] who provided the organizing thrust of today's member design, which later evolved into the Direct Strength Method (DSM) [10]. Hancock introduced the notion of the signature curve, from which quasi-pure buckling modes and associated loads could be determined, at least for typical design. Accessibility of FSM was enhanced by the introduction of the open source software CUFSM, first for pin-ended [11], but later for other end restraints [12]. Note that in all these FSM implementations (similarly to GBT) the primary loading is assumed to be longitudinal stresses, but recently FSM has also been applied to analyze shear buckling [13] of thin-walled members.

The mechanical criteria embedded in GBT led to the development of a special version of FSM, the constrained Finite Strip Method (*c*FSM), also implemented into CUFSM. *c*FSM possesses the ability of modal decomposition as well as mode identification in a manner similar to GBT [14–20]. In fact, *c*FSM and GBT have been extensively compared and found nearly coincident in their end results [21,22], even though the roots of the two methods are distinctly different.

Although *c*FSM can be considered as a theoretically interesting, and practically useful tool, it is not complete in its original form. *c*FSM is theoretically interesting, since it is the first method that provided a full and mechanics-based decomposition for a general deformation field. *c*FSM is practically useful, since it makes the design of thin-walled (especially cold-formed steel) members more straightforward and less subjective. However, *c*FSM is not complete, since it has three major limitations in its current development: (i) inherent limitations of FSM, (ii) limitations in the mechanical criteria used to separate the deformation classes, and (iii) currently only open cross-sections are covered.

A significant source of *c*FSM limitations derive from FSM itself. FSM requires that the member is flat-walled and prismatic, end restraints cannot be arbitrary, and discrete intermediate supports (along the length) cannot easily be handled. The requirement of prismatic sections, shared also by GBT, prevents direct application to tapered members, and members with holes. Recent works are aimed at partially removing these limitations, by applying *c*FSM base functions or GBT cross-section deformation modes to modal identification of shell FEM deformations fields [23–27], or by applying the constraining technique for spline FSM [28] or shell FEM [29,30], or by working out methods for the analysis of members with holes [31–35].

Another source of *c*FSM limitation is due to the mechanical criteria used for separation, the same criteria embedded in GBT. Specifically, although separation of D and L deformations is always possible, it leads to practically unsatisfactory results if the cross-section has small stiffeners or rounded corners. Recent proposals have been made on how to overcome these problems, at least for capacity prediction [36–38].

The third major source of current *c*FSM limitations is that closed cross-sections (or any cross-section with one or multiple closed parts) are excluded as originally proposed [14–17] and implemented into CUFSM [11,12]. Exclusion of closed cross-sections is partially due to the superficial handling of in-plane shear deformations (i.e., shear modes). The role of in-plane shear deformations are fully detailed in [39,40], and result in a new shear mode decomposition for *c*FSM. Careful implementation of the new shear modes is required for closed cross-sections, but also useful and practically meaningful for any cross-sections, and makes it possible to reproduce global buckling modes similar to those of shear-deformable beam theories. Since the original derivations only applied to open cross-sections generalization of the cross-section and incorporation of the new shear modes requires new notations and formulation for *c*FSM, as presented here.

In this paper, and its companion [41] the generalized *c*FSM is presented. In this paper the so-called primary modes are introduced: those deformation modes which are associated with minimal cross-section discretization (i.e. when nodal lines located at folds and ends only). In the companion paper the secondary modes are presented (i.e. those which exist only if flat plates are discretized into multiple strips), as well as some practical aspects are discussed and numerical examples are given.

2. Outline

2.1. FSM essentials

The finite strip method (FSM) is a shell-model-based discretization method. By utilizing longitudinal regularity, a common characteristic in thin-walled members, FSM requires a significantly smaller number of degrees of freedom (DOF) than the shell finite element method (FEM). Members are discretized into longitudinal strips as shown in Fig. 1. Note, in this paper (which focuses on primary modes only) only one strip per flat of the member is applied, as shown in Fig. 1. (Note, Fig. 1 illustrates the nodal displacements for the simplest longitudinal shape function as given in Eq. (2.4), with $m=1$.)

Within a strip, local displacement fields u, v, and w are expressed as follows [8]:

$$u(x,y)=\sum_{m=1}^{q}\begin{bmatrix}\left(1-\frac{x}{b}\right) & \left(\frac{x}{b}\right)\end{bmatrix}\begin{bmatrix}u_{1[m]}\\ u_{2[m]}\end{bmatrix}Y_{[m]} \tag{2.1}$$

$$v(x,y)=\sum_{m=1}^{q}\begin{bmatrix}\left(1-\frac{x}{b}\right) & \left(\frac{x}{b}\right)\end{bmatrix}\begin{bmatrix}v_{1[m]}\\ v_{2[m]}\end{bmatrix}Y'_{[m]}\frac{a}{m\pi} \tag{2.2}$$

$$w(x,y)=\sum_{m=1}^{q}\begin{bmatrix}\left(1-\frac{3x^2}{b^2}+\frac{2x^3}{b^3}\right) & \left(-x+\frac{2x^2}{b}-\frac{x^3}{b^2}\right) & \left(\frac{3x^2}{b^2}-\frac{2x^3}{b^3}\right) & \left(\frac{x^2}{b}-\frac{x^3}{b^2}\right)\end{bmatrix}\begin{bmatrix}w_{1[m]}\\ \vartheta_{1[m]}\\ w_{2[m]}\\ \vartheta_{2[m]}\end{bmatrix}Y_{[m]} \tag{2.3}$$

where a is the member length, and b is the strip width. For the simplest pin–pin end restraints the longitudinal shape functions are as follows:

$$Y_{[m]}=\sin\frac{m\pi y}{a} \quad \text{and} \quad Y'_{[m]}\frac{a}{m\pi}=\cos\frac{m\pi y}{a} \tag{2.4}$$