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# Numerical analysis of partly wrinkled cylindrical inflated beams under bending and shear

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ABSTRACT

This paper is aimed at assessing the nonlinear elastic response of an inflatable cylindrical beam through a simple mechanical model recently proposed by the authors for studying the equilibrium configurations of highly pressurised elastic membranes with general shapes. The attention is focused on beams loaded at mid-span with two different constraints, corresponding to simply-supported ends and built-in ends. The geometrical nonlinearities due to both the cross-sectional ovalization and wrinkling are carefully considered. In particular, the wrinkling of the membrane, clearly visible for load values much lower than the collapse load, is taken into account by means of an equivalent physical non-linearity. A two-states constitutive law for the material is assumed: when a fibre is stretched (the active state), its response is elastic, while when the fibre is contracted, no compressive force can be engendered in it (the passive state). The evolution of the cross-sectional ovalization, the size of the wrinkled regions and the magnitude of longitudinal and transverse stresses in the membrane are accurately determined for increasing levels of loads, up to collapse. The numerical results for the corresponding values of load and internal pressure, obtained through an expressly developed incremental-iterative algorithm, are compared with the experimental ones available in the literature.



## 1. Introduction

Inflated beams and arches made of highly pressurised very flexible membranes have seen a steadily enhancement in many engineering areas [1]. Properly fibre reinforced textile structures are a viable alternative to more traditional choices in all those cases in which the speed of execution or the lightness constitute primary requirements. In this regard it is enough considering, for example, all the emergency situations where the rapid set up and final removal of shelters is of crucial importance, or the structures designed to operate in the space, which have to be as lightweight and easily portable as possible.

Despite the importance and the diffusion of these elements, some basic aspects characterising their mechanical response have not yet sufficiently cleared and may therefore be placed among the current topics of mechanics of structures [2]. No doubt the uncertainties arising in the study of the mechanical behaviour of inflated elements mainly come from the strong nonlinear phenomena, either material or geometrical, inherent in membranes.

Constitutive laws for coated woven fabrics under biaxial tensile loading very frequently proved to be nonlinear in the usual working range. The highly nonlinear response is evident in the typical load–extension diagram of both the yarn fibres and the fabric coating [3]. Moreover, in inflated structures the mechanical behaviour of plain-woven fabrics exhibits a highly nonlinear dependence on the internal pressure [4,5]. Conversely, geometrical non-linearities characterise the response of inflated beams at different levels. Speaking in broad terms, *global* non-linearities usually concern the beam in its entirety. The relatively small stiffness of the beam under shear, bending, torsion or axial loads allows for large displacements and rotations, so that the inflated equilibrium configuration is usually far from the initial one. At the *local* level, the nonlinear behaviour is promoted by the cross-sectional ovalization, which mainly takes place in the neighbourhood of the point of application of the loads. Finally, due to the smallness of membrane thickness values (either the actual one or an equivalent one, if the wall is made of some structural fabric), the *ultra-local* geometric nonlinearities corresponding to wrinkling phenomena are likely to appear. Wrinkling may interest large portions of the membrane's surface; moreover, with increasing loads, the size of the corrugated zones, whose position constitutes one of the main unknowns of the problem, tends to increase, thus

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strongly influencing the overall behaviour of the whole element and, what it is most important, its actual carrying capacity.

At present, it has not yet been completely cleared how the global, local and ultra-local nonlinear phenomena affect each other and the overall mechanical response of the beam as well. A solid understanding of this topic is of basic importance whenever a reliable prediction of the behaviour of the inflated beam in proximity of the ultimate load is needed.

In the literature, several works have been addressed to the modelling of the load–deflection behaviour of circular cylindrical inflated beams. In a first line of research, inflated beams are considered as thin shells. Among the many contributions, we recall here the first pioneering work published by Brazier [6]. In it, he derived the collapse bending moment by minimising the shell strain energy. The ovalization of the originally circular cross-section was correctly accounted for as well. Linear elasticity and calculus of variation tools were used. The same problem of a cylindrical shell under bending has been subsequently considered, among others, in [7,8] and, in recent years, in [9].

In a second, more recent, line of reasoning the wall of the inflated beam is modelled as an ideal membrane without any bending and torsional stiffness. Starting from the work by Stein and Hedgepeth in 1961 [10], where an approximated solution is obtained by properly modifying the constitutive relations for the material, many other contributions followed [11–13], . A step forward in modelling inflated membranes has been made by Wu [14], who first considered wrinkles as an inelastic strain, and by Pipkin [15], who introduced suitable relaxed strain energy expressions accounting for wrinkling. Many researchers followed and developed these ideas (in this regard see, as an example, [16,17]).

In this paper, we aim at investigating the mechanical response of inflatable beams under bending and shear. Section 2 gives a short presentation of the proposed new model, which accounts for large displacements and rotations, cross-sectional ovalization and wrinkling of the membrane. In particular, we assess the state of stress in the wall of an inflatable cylindrical beam through a simple mechanical model recently proposed by the authors to study the equilibrium shapes of highly pressurised elastic membranes [18]. The aforesaid local geometric nonlinearities are taken into account by means of an equivalent physical non-linearity, assuming a two-states constitutive law for the material: when a fibre is stretched (the active state), its response is elastic, while when the fibre is contracted, no compressive force can be engendered in it (the passive state). In the same section, the expressly developed numerical incremental-iterative solution method is described. In Sections 3 and 4 the mechanical response of the inflated beam in the two basic cases corresponding to simply-supported and built-in ends, respectively, is illustrated. In each of the study case, the evolution of the wrinkled regions and the

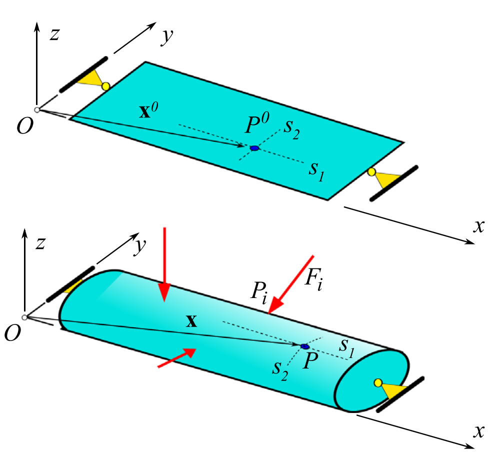


**Fig. 1.** Symbols and notations.

distribution of longitudinal and transverse stresses in the membrane are determined accurately for increasing levels of loads, up to collapse. The effects produced by the cross-sectional ovalization and wrinkling phenomena on the overall response of the inflated beam are accurately analysed. The role of the internal pressure is discussed. Lastly, the numerical results are compared with the numerical and experimental ones available in the literature.

## 2. A mechanical model for an inflatable beam element

In the following, we will focus our attention on the bending response of a cylindrical inflatable beam, variously supported at its ends, subjected to a transversal concentrated load acting at mid-span. The inflated element is schematised as a closed membrane where a two-dimensional positive semi-definite state of stress may take place. As it will be shown, the model allows for finite displacements, cross-sectional ovalization and wrinkling phenomena.

### 2.1. Problem formulation

For our aims, we model the beam element as a close elastic membrane inflated by an inner pressure. We assume that in the initial, unstrained configuration the membrane is flat, and that two overlapping plane thin films of small thickness $t$ compose it. The loaded membrane may undergo large displacements/rotations, but only small or moderate strains, while a nonlinear elastic constitutive law that makes uses of a relaxed energy allows for accounting for possible wrinkling of the membrane.

We restrict our attention to problems symmetrical with respect to the plane the membrane initially belongs to. Because of the aforesaid symmetry, we consider only the upper half of the membrane. We adopt a total Lagrangian formulation.

We indicate with $(s_1, s_2)$ the internal coordinates of any given point on the membrane, and with $\mathbf{x}^0(s_1, s_2)$ and $\mathbf{x}(s_1, s_2)$ the position vector of the same point in the initial and current configuration, respectively (Fig. 1). The strain gradient $\mathbf{F}$ and the Green's strain tensor $\mathbf{E}$ are given by the standard relations:

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{x}^0}, \quad \mathbf{E} = \frac{1}{2}(\mathbf{F}^{\mathrm{T}}\mathbf{F} - \mathbf{I}), \tag{1}$$

where the superscript T denotes the matrix transpose operation.

Following an idea firstly introduced by Wu [14], we assume that the overall strain $\mathbf{E}$ may be decomposed into the sum of an elastic quote, $\mathbf{E}^e$, and an inelastic quote, $\mathbf{E}^w$, named *wrinkle strain*

$$\mathbf{E} = \mathbf{E}^e + \mathbf{E}^w \tag{2}$$

Elastic quote $\mathbf{E}^e$ alone is responsible for the stress state, whereas wrinkle strain $\mathbf{E}^w$ is an additional negative semi-definite inelastic strain that may be though as related to the out-of-plane component of the displacement of points belonging to wrinkled regions of the surface. The original idea by Wu has been resumed and extended to finite elasticity by many researches during the years that followed its first introduction (by way of example, the interested reader may refer to [19]). The introduction of the wrinkle strain corresponds to some kind of "smearing" of the wrinkles on the average mid-surface of the membrane (the so-called "*pseudo-deformed*" surface). Such a smearing operation can be considered having an acceptable approximation in the limit case where the membrane's thickness goes to zero, or alternatively the tensile stress goes to infinity.

A linear elastic constitutive law between elastic strain $\mathbf{E}^e$ and the work-conjugated Piola-Kirchhoff stress tensor, $\mathbf{S}$, is assumed for the material:

$$\mathbf{S} = C\,\mathbf{E}^e = C\,(\mathbf{E} - \mathbf{E}^w), \tag{3}$$

where C represents the usual fourth-order elasticity tensor. Moreover, we postulate by hypothesis the following conditions to hold