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The prediction of the elastic critical load of submerged elliptical cylindrical shell based on the vibro-acoustic model

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A B S T R A C T

Based on the vibro-acoustical model, an effective new approach to nondestructively predict the elastic critical hydrostatic pressure of a submerged elliptical cylindrical shell is presented in this paper. Based on the Goldenveizer–Novozhilov thin shell theory, the vibration equations considering hydrostatic pressures of outer fluid are written in the form of a matrix differential equation which is obtained by using the transfer matrix of the state vector of the shell. The fluid-loading term is represented as the form of Mathieu function. The data of the fundamental natural frequencies of the various elliptical cylindrical shells with different hydrostatic pressure and boundary conditions are obtained by solving the frequency equation using Lagrange interpolation method. The curve of the fundamental natural frequency squared versus hydrostatic pressure is drawn, which is approximately straight line. The elastic critical hydrostatic pressure is therefore obtained while the fundamental natural frequency is assumed to be zero according to the curve. The results obtained by the present approach show good agreement with published results.

1. Introduction

Cylindrical shells are widely used in various underwater and marine structures. Since the elastic buckling load plays a very important role in the safety of those structures, numerous researchers have investigated the elastic buckling pressure of cylindrical shell with various methods [1–3]. In the past, the nondestructive prediction approach of the buckling loads also has been studied by many authors with experiments and FEM simulations. However, simulation analysis [4–6] needs heavy workload, while experimental method [7,8] is costly and time-consuming. In the view of the disadvantages of the two methods, theoretical method has been widely used recently [9,10]. Plaut [11] revealed the relationship between square of the fundamental natural frequency and critical load. Then they forecast the upper and lower bound of critical load of cylindrical shell. Recently, Zhu et al. [12] have proposed a new analytical method to nondestructively predict the elastic critical pressure of a submerged cylindrical shell based on wave propagation method.

Those investigations mainly focus on the elastic buckling of circular cylindrical shells. However, the elliptical cylindrical shells are quite common in the practical applications due to the manufacturing tolerance [13] and other special manufacturing requirements [14]. Since the curvature of cross section of the elliptical shell is unevenly distributed, both the theory and method which were used to study the circular cylindrical shell problems cannot be directly applied to elliptical shell due to its cross section is noncircular. Also, because of the particular geometry characteristics of the cross section, greater mathematical challenges are posed and closed-form analytic solutions cannot be obtained, numerical or approximate techniques are necessary to deal with the problem of elliptical cylindrical shells. Researches on elliptical cylindrical shells are much less than those on cylindrical shells. Moreover, there have been very few isolated analytical studies on the vibro-acoustic model of elliptical cylindrical shells because the solution of wave equation in elliptical coordinate are much complex than that in cylindrical coordinates. The only scanty papers which involve the vibro-acoustic model of elliptical cylindrical shells [15,16] present a limited discussion and were focused on the sound propagation in the elliptical ducts. Due to the above reasons, researches on the elastic buckling of submerged elliptical cylindrical shells based on vibro-acoustic model are seldom reported.

We intend to address this challenge by developing the vibro-acoustic model for elliptical cylindrical shell structures subject to hydrodynamic loading to predict the elastic critical load. In this paper, the hydrostatic pressure of outer fluid is considered as an external load imposed on the shell. Then, the vibration equations containing the hydrostatic pressure term are written in a matrix differential equation by using the transfer matrix. The fluid-loading

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term is represented as the form of Mathieu function. The natural frequencies are obtained by solving the frequency equations which is dependent on the boundary conditions. The elastic critical pressure is obtained by fitting the curve of the fundamental frequency squared versus hydrostatic pressure with straight line. The results of the degradation model that obtained by the present method agree pretty well with those of existing literatures.

2. Vibration equations of the shell

2.1. System description

A thin-walled, finite length elliptical cylindrical shell with density ρ, Young's modulus E and Poisson's ratio v is shown in Fig. 1. The semi-major and semi-minor axes are a and b respectively. The length and thickness of the shell is L and h, respectively. The axial, circumferential and normal displacements of the shell are represented by u, υ, w respectively. The shell is referred to an elliptic coordinates (ξ, η, z). R is the variable radius of curvature which is related to coordinate η and is defined as $R=R(\eta)$. r_0 is the radius of a circular cylindrical shell which has the same perimeter as that of the cross section of the elliptical cylindrical shell and it means an average radius of the noncircular cross section.

According to Fig. 1, elliptic coordinates and Cartesian coordinates have following relations:

$$x=f\cosh\xi\cos\eta \tag{1a}$$

$$y=f\sinh\xi\sin\eta \tag{1b}$$

$$z=z \tag{1c}$$

where f is the focal length,

$$f=\sqrt{a^2-b^2} \tag{2}$$

The outer surface of the cross section is defined by $\xi=\xi_0$ and the geometrical relation between ξ_0 and the semi-major axis a and the semi-minor axis b is

$$\xi_0=\operatorname{arctanh}(b/a) \tag{3}$$

According to Eqs. (1a) and (1b), each point on the outer surface of the cross section in the Cartesian coordinates can be written as

$$x=f\cosh\xi_0\cos\eta \tag{4a}$$

$$y=f\sinh\xi_0\sin\eta \tag{4b}$$

From Eqs. (4a) and (4b), the following equations can be derived immediately:

$$dx=-f\cosh\xi_0\sin\eta d\eta \tag{5a}$$

$$dy=f\sinh\xi_0\cos\eta d\eta \tag{5b}$$

The arc length element ds of the outer surface of the cross section along the circumferential direction is given by

$$ds=\sqrt{dx^2+dy^2} \tag{6}$$

Substituting Eq. (5) into Eq. (6), it follows that

$$ds=f\sqrt{\cosh^2\xi_0\sin^2\eta+\sinh^2\xi_0\cos^2\eta}\,d\eta \tag{7}$$

For convenience, denoting that

$$\phi(\eta)=f\sqrt{\cosh^2\xi_0\sin^2\eta+\sinh^2\xi_0\cos^2\eta} \tag{8}$$

Now, Eq. (7) can be rewritten in a compact form:

$$ds=\phi(\eta)d\eta \tag{9}$$

2.2. Governing equations

For a submerged elliptical cylindrical shell, the outer hydrostatic pressure, p_0, can be divided into a uniform normal pressure on the shell wall and an axial compression applied at the two edges. The respective circumferential force N_η^0 and the axial force N_z^0 are

$$N_z^0=p_0ab/2r_0 \tag{10a}$$

$$N_\eta^0=Rp_0 \tag{10b}$$

According to the Goldenveizer–Novozhilov thin shell theory [17,18], the vibration equations of a submerged elliptical cylindrical shell are written as

$$\frac{\partial N_z}{\partial z}+\frac{1}{\phi}\frac{\partial N_{\eta z}}{\partial \eta}-N_\eta^0\left(\frac{1}{\phi}\frac{\partial^2 \upsilon}{\partial z\partial\eta}+\frac{1}{R}\frac{\partial w}{\partial z}\right)+\rho h\omega^2u=0 \tag{11a}$$

$$\frac{\partial N_{z\eta}}{\partial z}+\frac{1}{\phi}\frac{\partial N_\eta}{\partial \eta}+\frac{Q_\eta}{R}+N_z^0\frac{\partial^2 \upsilon}{\partial z^2}+\rho h\omega^2\upsilon=0 \tag{11b}$$

$$\frac{\partial Q_z}{\partial z}+\frac{1}{\phi}\frac{\partial Q_\eta}{\partial \eta}-\frac{N_\eta}{R}-\frac{1}{\phi}N_\eta^0\left(\frac{1}{R}\frac{\partial \upsilon}{\partial\eta}-\frac{1}{\phi}\frac{\partial^2 w}{\partial \eta^2}\right)+N_z^0\frac{\partial^2 w}{\partial z^2}+\rho h\omega^2 w=P \tag{11c}$$

$$\frac{\partial M_z}{\partial z}+\frac{1}{\phi}\frac{\partial M_{\eta z}}{\partial \eta}-Q_z=0 \tag{11d}$$

$$\frac{\partial M_{z\eta}}{\partial z}+\frac{1}{\phi}\frac{\partial M_\eta}{\partial \eta}-Q_\eta=0 \tag{11e}$$

$$S_\eta-Q_\eta-\frac{\partial M_{\eta z}}{\partial z}=0 \tag{11f}$$

where P is the dynamic pressure of the fluid acting on the shell, whereas N_z, N_η and Q_z, Q_η are the normal and transverse shear

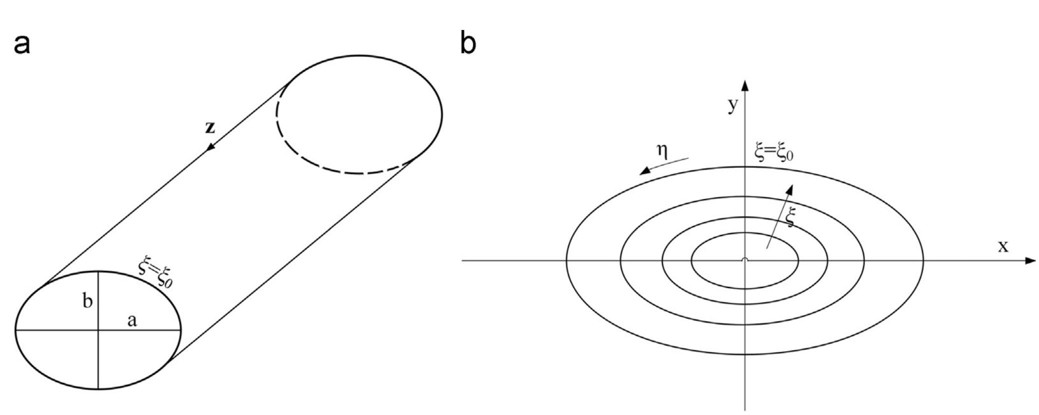

Fig. 1. Geometry and coordinates of the elliptical cylindrical shell. (a) Coordinates, and (b) elliptical cylindrical shell.