Thin-Walled Structures 84 (2014) 325–334

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

# On the shear deformation modes in the framework of Generalized Beam Theory

Rodrigo Gonçalves [a,*], Rui Bebiano [b], Dinar Camotim [b]

[a] UNIC, Departamento de Engenharia Civil, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal
[b] DECivil, ICIST, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisbon, Portugal

ARTICLE INFO

*Article history:*
Received 24 June 2014
Received in revised form
15 July 2014
Accepted 16 July 2014
Available online 7 August 2014

*Keywords:*
Thin-walled members
Generalized Beam Theory (GBT)
Cross-section deformation
Shear deformation
Shear flow

ABSTRACT

This paper extends previous work concerning the determination of cross-section deformation modes in thin-walled members with arbitrary polygonal cross-section, in the framework of Generalized Beam Theory (Gonçalves et al., 2010 [1]). In particular, the paper addresses the so-called “natural shear deformation modes” (i.e. the deformation modes that involve non-null membrane shear strains and are independent of the cross-section discretization employed), which are relevant for capturing the behaviour of thin-walled members with complex multi-cell cross-sections undergoing torsion and/or distortion. The contributions of the paper are (i) the derivation of fundamental properties of the shear modes, (ii) the proposal of an efficient mode extraction procedure and (iii) the development of analytical results for several particular cases. In order to illustrate the application of the proposed mode extraction procedure and demonstrate the validity of the derived properties, several cross-sections are analyzed, including complex multi-cell tubes.



## 1. Introduction

Generalized Beam Theory (GBT), introduced by Schardt [2], is a thin-walled beam theory that is capable of capturing cross-section in-plane and out-of-plane (warping) deformation through the inclusion of additional degrees-of-freedom, the so-called “cross-section deformation modes”. GBT has been continuously developed and it has been widely demonstrated that it is capable of providing very accurate and computationally efficient solutions for a wide range of structural problems involving prismatic thin-walled members (see, e.g., www.gbt.infofor a list of earlier publications and [3–6] for the description of recent developments by Camotim and co-workers). In particular, the intrinsic modal decomposition features of GBT provide in-depth information concerning the mechanics of the problem addressed and make it possible to derive analytical or semi-analytical formulas with a wide range of application.

In a recent paper [1], a general approach for calculating the deformation modes for arbitrary polygonal cross-sections was proposed, focusing specifically on issues associated with the enforcement of kinematic constraints, such as those resulting from the imposition of null strain components (e.g., null membrane shear strains) and the presence of external/internal restraints (e.g., cross-section symmetry simplifications, box diaphragms). These constraints constitute a key aspect of most GBT formulations, as they can effectively reduce the number of admissible deformation modes (i.e., DOFs) without sacrificing accuracy. In fact, it is shown in [1] that, in many cases, rather than employing the complete set of deformation modes, it is more efficient to supplement the so-called “conventional modes” with a few modes allowing for shear and/or transverse extension deformation in relevant parts of the cross-section – for instance, closed sections require cell shear flow modes and wide flange sections require shear lag modes. Nevertheless, it should be mentioned that the enforcement of the constraints is rather challenging for complex cross-sections, since not all cross-section nodes/walls may undergo independent displacements (precisely the motivation behind [1]).

This paper extends the previous work by exploring further the properties of the so-called “natural shear deformation modes”, i.e., the deformation modes that (i) involve non-null membrane shear strains, (ii) comply with the null membrane transverse extension assumption (i.e., the walls are deemed inextensible in the cross-section plane) and (iii) are independent of the GBT cross-section discretization employed. As shown in [1], these modes are particularly relevant for closed cross-sections undergoing torsion and distortion. The new developments are the following: (i) the fundamental properties of the natural shear modes are derived and discussed (Section 3), (ii) analytical results for several cross-section

* Corresponding author. Tel.: +351 21 2948580; fax: +351 21 2948398.
*E-mail address:* rodrigo.goncalves@fct.unl.pt (R. Gonçalves).

http://dx.doi.org/10.1016/j.tws.2014.07.012
0263-8231/

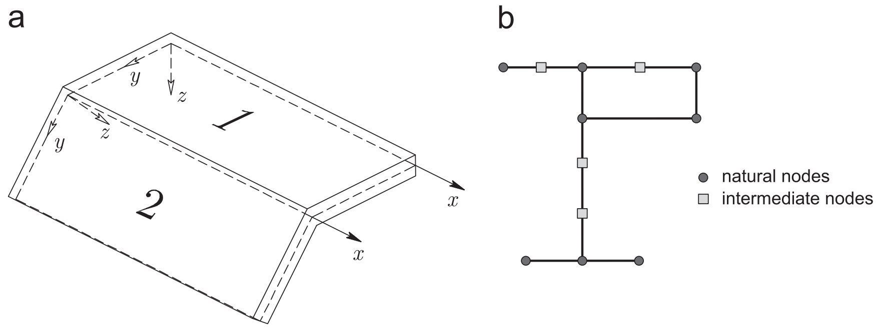


**Fig. 1.** (a) Arbitrary thin-walled member local coordinate systems. (b) Cross-section discretization

types are provided (Section 4) and (iii) a computationally efficient procedure for their extraction is proposed (Section 5). Moreover, Section 6 presents a set of examples to illustrate the application of the mode extraction procedure and demonstrate the validity of the derived properties. In particular, representative cross-sections are analyzed, including complex multi-cell tubes.

The notation presented in [1] is followed and a summary of the main results of this paper is provided in Section 2. For simplicity, it is assumed that no external/internal cross-section restraints exist, since these can be efficiently enforced at the structural analysis level, using appropriate constraint [7,8]. Nevertheless, the procedures and results presented in this paper can handle quite easily these restraints, using the concepts introduced in the previous paper.

## 2. Summary of previous results

According to the usual GBT kinematic description, with the wall mid-surface local axes $(x, y, z)$ shown in Fig. 1(a), the displacement vector $\mathbf{U}$ is given by

$$\mathbf{U}(x,y,z)=\begin{bmatrix}U_x\\U_y\\U_z\end{bmatrix}=\begin{bmatrix}u(x,y)-zw_{,x}(x,y)\\v(x,y)-zw_{,y}(x,y)\\w(x,y)\end{bmatrix}, \tag{1}$$

where the commas indicate differentiation and $u, v, w$ are the wall mid-surface displacement components along $x, y, z$, respectively, which are approximated through

$$u(x,y)=\sum_{k=1}^{D}\overline{u}_k(y)\phi_{k,x}(x), \tag{2}$$

$$v(x,y)=\sum_{k=1}^{D}\overline{v}_k(y)\phi_k(x), \tag{3}$$

$$w(x,y)=\sum_{k=1}^{D}\overline{w}_k(y)\phi_k(x), \tag{4}$$

where subscript $k=1,\ldots,D$ indicates the cross-section deformation mode number, with shape functions $\overline{u}_k(y)$, $\overline{v}_k(y)$, $\overline{w}_k(y)$ and amplitude function along the beam length $\phi_k(x)$.

The functions $\overline{u}_k, \overline{v}_k, \overline{w}_k$ are obtained through the so-called "cross-section analysis", which comprises the following steps:

1. *Cross-section discretization*: Fig. 1(b) shows a typical discretization, which involves (i) "natural" nodes, automatically located at wall mid-line intersections and free edges, and (ii) "intermediate" nodes, which are arbitrarily located between the natural nodes and define the discretization level.
2. *Calculation of an initial deformation mode set*: The initial modes are obtained on the basis of the nodal DOFs and two mode subspaces are defined: (i) the "natural" subspace $\mathcal{N}$, involving natural node DOFs, with intermediate node DOFs constrained out (thus these modes are independent of the cross-section discretization), and (ii) the "local" subspace $\mathcal{L}$, whose modes do not involve natural node DOFs and, thus, depend on the cross-section discretization.
3. *Calculation of the final modes*: The final mode set is obtained through change of basis operations (sometimes called mode orthogonalization operations), using generalized eigenvalue problems. As discussed in [1], several possibilities exist. The procedure for the conventional modes has been well-established in [2].

Since $\mathcal{L}$ is completely dealt with in [1], it is not addressed in the present paper and, consequently, no intermediate nodes are considered. Moreover, as customary in most GBT formulations, the initial deformation mode set is generated (i) using linear $\overline{u}$ (warping) functions between nodes and (ii) calculating the in-plane functions $\overline{v}, \overline{w}$ by analyzing the cross-section as a plane frame, subjected to imposed displacements.

Attention is now focused on the main results obtained in [1] concerning the natural mode subspace $\mathcal{N}$, which constitute the foundations of the present work. It can be shown that $\mathcal{N}$ may be subdivided according to

$$\mathcal{N}=\mathcal{N}_{v+w}^{\varepsilon_y\neq 0}\cup\mathcal{N}_{v+w}^{\varepsilon_y=0}\cup\mathcal{N}_u, \tag{5}$$

where the subscripts identify the displacement components involved and the superscripts indicate whether the walls are assumed inextensible along $y$ or not (i.e., if $\varepsilon_{yy}^M$ is null or not). Furthermore,

$$\mathcal{N}_{v+w}^{\varepsilon_y=0}=\mathcal{N}_v\cup\mathcal{N}_w, \tag{6}$$

where $\mathcal{N}_v$ involves the independent $v$ displacements of the walls and $\mathcal{N}_w$ concerns $w$ nodal displacements at outstands (wall free ends). It turns out that $N_v=\dim(\mathcal{N}_v)$ may be smaller than the number of walls ($n_{walls}$), namely

$$N_v+N_v^*=n_{walls}, \tag{7}$$

where $N_v^*$ is the number of "dependent walls". Furthermore, except for cross-sections with a single wall or radiating walls, $\mathcal{N}_w$ may be constrained out – this is the approach adopted in the present paper.

The combination of $\mathcal{N}_v$ and $\mathcal{N}_u$ may be decomposed in the following manner:

$$\mathcal{N}_v\cup\mathcal{N}_u=\mathcal{N}_\gamma\cup\mathcal{N}_{\text{Vlasov}}, \tag{8}$$

where $\mathcal{N}_\gamma$ designates the natural shear mode space (with non-null membrane shear strains and null membrane transverse extensions[1]) and $\mathcal{N}_{\text{Vlasov}}$ designates the so-called Vlasov mode space,

[1] In [1], this mode space is designated as $\mathcal{N}_{\gamma\neq 0}^{\varepsilon_y=0}$, but the notation is simplified in the present paper.