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Re-interpreting simultaneous buckling modes of axially compressed isotropic conical shells

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ABSTRACT

Elastic stability of shell structures under certain loading conditions is characterized by a dramatically unstable postbuckling behavior. The presence of simultaneous 'competing' buckling modes (corresponding to the same critical buckling load) is understood to be largely responsible for such behavior. In this paper, within the framework of linear bifurcation eigenvalue analysis and Donnell shallow shell theory, the presence of simultaneous buckling modes in axially compressed isotropic cones is determined using the semi-analytical method of Galerkin. The results are presented in the plane of the dimensionless reciprocal meridional and circumferential buckling half wavelengths, and are compared with the locus of simultaneous buckling modes of axially compressed cylinders, described by the so-called *Koiter circle*. By using an optimizing procedure, it is shown that the cluster of simultaneous buckling modes in cones is well described by the Koiter circle of an equivalent cylinder with appropriate length and radius. Such optimizing values of length and radius allow us to gain some insight into the simplifying treatment of the buckling of cones through the concept of equivalent cylinder.

1. Introduction

Buckling is one of the most important failure modes in thin-walled shell structures submitted to compressive stresses. Indeed a large amount of research work has been carried out in the past decades to determine the buckling strength of shell structures [1,2]. The simple geometry of cylinders has received a much larger attention in the literature in comparison to other shell shapes [3–6].

It is well known from experimental and theoretical studies that in shell structures the actual failure load might be significantly lower than that obtained from a linear bifurcation eigenvalue analysis, even if the material fails in its elastic range [6]. One of the reasons for such a reduction is attributed to the presence of local or global geometric imperfections which produce a deviation from the nominal geometry of the shell. The early fundamental works by von Kármán and Tsien [7], Donnell and Wan [8] and Koiter [9] demonstrated how initial geometric imperfections can be a primary source of variation between analytical predictions and experimental results. It is known that the membrane component of strain energy is the most important factor in imperfection sensitivity of the shell [10]. Axial compression is the loading condition allowing the shell to develop a high membrane component of the total strain energy, which will result in higher imperfection sensitivity. Thus, this loading condition is largely investigated in cylindrical shells [11–15]. The high imperfection sensitivity is often explained by the presence of simultaneous – interacting on each other – buckling modes corresponding to the same critical buckling load.

In addition to cylinder, other shells of revolution such as cone and sphere can be brought to a high membrane stress state. This membrane stress can be achieved in cylinders and cones through axial compression and internal pressure and for spheres through pressurization. Buckling of axially compressed conical shells has received relatively much less attention in the literature. There is however similarities in the buckling behavior of conical shells with respect to that of cylinders. For instance, former investigations proved that conical shells are imperfection sensitive when exposed to axial compression (Lackman and Penzien [16]; Spagnoli [17]; Chryssanthopoulos et al. [18]). In addition, the analytical expression of the buckling load of conical shells under axial compression is correlated to that of cylindrical counterparts. Seide [19] presented a simple closed form solution for buckling of axisymmetric conical shells as

$$P_{\text{cone}} = \frac{2\pi E t^2}{\sqrt{3(1-\nu^2)}}\cos^2\beta = P_{\text{cyl}}\cos^2\beta \tag{1}$$

where E and ν are the Young modulus and Poisson ratio of the material, respectively, t is the thickness of the shell and β is the

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semi-vertex (tapering) angle of the cone as shown in Fig.1. This expression is based on the classical membrane prebuckling state assumption and shallow shell theory of Donnell.

In the design against buckling of conical shells, it is a common practice to treat the cone as an equivalent cylinder. This so-called equivalent cylinder concept simply originates from the expression of the membrane stress related to the critical buckling load at a given location defined by parallel radius r, namely

$$\sigma_{x,cr} = \frac{P_{\text{cone}}}{2\pi rt\ \cos\beta} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{t\ \cos\beta}{r} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{t}{r_e} \tag{2}$$

where r_e is the radius of curvature of the cone at the given location. It is evident from Eq. (2) that the buckling membrane stress along the meridian of the cone can be regarded as that of an equivalent cylinder with equal thickness having the radius equal to the corresponding radius of curvature of the cone. In other words, the radius r_e of the equivalent cylinder is such that the critical buckling resistance of the equivalent cylinder is equal to that of the conical shell [20].

The formal similarity of the expressions for the buckling strength of cylinders and cones has suggested, at least as long as the tapering angle of the cone is not too high (say, less than 65°) – otherwise a snap-through instability type might be dominant [20], to extend such a similarity to cones' bifurcation behavior in general as well as to their imperfection sensitivity under axial compression and other loading conditions. This leads to the definition, for both buckling strength and imperfection knock-down calculations, of specific values of the radius r_e as well as of the length l_e of the equivalent cylinder as a function of the type of load acting on the conical shell [20]. The equivalent cylindrical concept was confirmed by many researchers such as Esslinger and Ciprian [21], Pariatmono and Chryssanthopoulos [22] and Schmidt and Krysik [23] for elastic and also Blachut [24] for elastic–plastic behavior of conical shells under axial compression.

All above-mentioned attempts are limited in finding length and radius of the equivalent cylinder so as to obtain a matching between the critical buckling stress of the equivalent cylinder and that of the cone at a certain location, but they do not contain discussions about simultaneous buckling modes of the cone and of the corresponding equivalent cylinder. As a matter of fact, it might be conjectured that if an agreement between cone and equivalent cylinder can be found not only in terms of critical stress but also in terms of simultaneous mode distribution, the imperfection sensitivity of the cone can be described by that related to the equivalent cylinder.

An attempt to explore the presence of simultaneous buckling modes in cones has been presented by Poggi [25]. Pariatmono and Chryssanthopoulos [22] showed that at a certain aspect ratio of an axially compressed conical shell, different buckling modes correspond to the same value of critical stress and Spagnoli [26] used finite element method to show that the locus of these modes is described by an ellipse whose aspect ratio is dependent on the tapering angle of the cone.

In this paper, within the framework of linear bifurcation eigenvalue analysis and Donnell shallow shell theory, the presence of simultaneous buckling modes in axially compressed cones is determined using the semi-analytical method of Galerkin. The results are presented in the plane of the dimensionless meridional and circumferential buckling half wavelengths, and are compared with the locus of simultaneous buckling modes of axially compressed cylinders, described by the so-called Koiter circle. By using an optimizing procedure, it is shown that the cluster of simultaneous buckling modes in cones is well described by the Koiter circle of an equivalent cylinder with appropriate length l_e and radius r_e. Such optimizing values of length and radius allow us to gain some insight into the simplifying treatment of the buckling of cones through the concept of equivalent cylinder.

2. Geometry and definitions

Consider a conical shell with the (x, θ, z) coordinate system shown in Fig. 1, where x is the coordinate along the cone generator, θ is the circumferential coordinate and z is the coordinate normal to the cone surface as shown in Fig. 1. The radius at the small and large end is r_1 and r_2, respectively, β is the tapering angle of cone and L is the slant length of the cone along the generator. The thickness of the cone is t and r_0 represents its mean radius.

3. Doubly periodic buckling modes and the Koiter circle of cylindrical shells

A brief survey on Koiter circle in cylindrical shells under uniform axial compression is presented in this section. A detailed discussion can be found in Section 14.3 of Calladine [27] and Spagnoli [26]. The governing stability equation of Donnell for axially compressed cylinders, neglecting in-plane deflections, can be written as follows [28]

$$D\nabla^4(\nabla^4 w) + \frac{1-\nu^2}{r^2}Cw_{,xxxx} + \frac{P}{2\pi r}\nabla^4 w_{,xx} = 0 \tag{3}$$

where w is the incremental out-of-plane displacement with respect to the initial (prebuckling) state, x is the coordinate along the cylinder length, θ is the circumferential coordinate, the comma subscript corresponds to partial differentiation against the variable indicated. The differential operator is

$$\nabla^4(\,) = (\,)_{,xxxx} + \frac{2}{r^2}(\,)_{,xx\theta\theta} + \frac{1}{r^4}(\,)_{,\theta\theta\theta\theta} \tag{4}$$

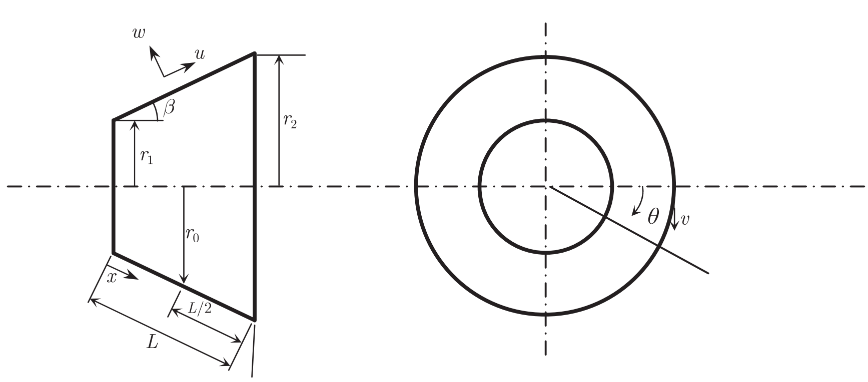

Fig. 1. Geometry of the cone.