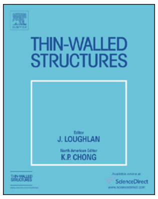
Thin-Walled Structures 84 (2014) 406–415

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws



An isogeometric approach for the analysis of composite steel–concrete beams

M. Lezgy-Nazargah*

Faculty of Civil Engineering, Hakim Sabzevari University, Sabzevar, Iran

ARTICLE INFO

Article history:
Received 20 March 2014
Received in revised form
15 July 2014
Accepted 16 July 2014
Available online 13 August 2014

Keywords:
Isogeometric analysis
NURBS
Composite steel–concrete beams
Refined high-order theory
Transverse shear and normal stresses

ABSTRACT

An isogeometric approach based on non-uniform rational B-spline (NURBS) basis functions is presented for the analysis of composite steel–concrete beams. A refined high-order theory is considered in deriving the governing equations using the principle of virtual work. The employed theory satisfies all the kinematic and stress continuity conditions at the layer interfaces and considers effects of the transverse normal stress and transverse flexibility. The global displacement components, described by polynomial or combinations of polynomial and exponential expressions, are superposed on local ones chosen based on the layerwise concepts. The present isogeometric formulation does not need incorporating any shear correction factor. Moreover, in the present isogeometric formulation, the number of unknowns is independent of the number of layers. The proposed isogeometric formulation is validated by comparing the present results with the available published and the three-dimensional (3D) finite element results. In addition to correctly predicting the distribution of all stress components of the composite steel–concrete beams, the proposed formulation is computationally very economic.

1. Introduction

Much attention has been paid on composite steel–concrete structures in recent years due to their economical and structural advantages. Composite steel–concrete beams are the most common applications of the composite structures which are typically made of concrete slabs cast on steel members. Taking the advantage of the concrete in compression and steel in tension, composite steel–concrete beams have enhanced stiffness and strength performances in comparison to each components considered in isolation.

Various models have been presented in the literature to date for the analysis of composite steel–concrete beams [1]. Most of these earlier studies are based on Euler–Bernoulli's beam theory (EBT). Although the Euler–Bernoulli hypothesis is very successful in the prediction of global responses (e.g. deflection, fundamental natural frequency or buckling load) of slender composite beams, it fails in the prediction of local behaviors (e.g. through the thickness distribution of transverse shear and normal stresses, high natural frequencies) due to the neglecting of the transverse shear stress. Moreover, the transverse shear stress affects on the global response of beams having small length to thickness ratio, low shear rigidity or continuous spans [2]. In order to incorporate the effect of shear stress in the analysis, some researchers [3–5] used Timoshenko's beam theory (TBT) for the analysis of composite steel–concrete beams. Ranzi and colleagues [6–9] used a combination of EBT and TBT to analyze composite steel–concrete beams. They used EBT to model the concrete slab while the steel girders were modeled using TBT. Although TBT leads to more accurate results in comparison to EBT, it gives a uniform shear stress distribution over the beam thickness whereas the actual variation of shear stress is parabolic. In order to remove this drawback of TBT, some researchers modified the shear stiffness of the beams by employing a shear correction factor. This factor which is dependent on the cross-sectional area of the beam, has a different value depending on the geometry and material properties of the beam. Under the action of static loads, Whitney [10] evaluated shear correction factors of multilayered rectangular composite laminated beams. However, an accurate estimation of the shear correction factor is a complex procedure. Moreover, an analysis based on TBT cannot accurately predict the local responses of composite beams especially the distribution of the transverse shear and normal stresses. In order to overcome these limitations, several high-order beam theories (HBTs) have been presented in the literature to date. These high-order theories contain the third-order shear deformation theory [11], layer-wise or discrete-layer theories [12–15], zig–zag theories [16–21], global–local theories [22–25] and mixed theories [26,27]. Although high-order theories are able to predict the local responses of composite beams accurately, they are often used for the analysis of multilayered

---

* Tel.: +98 571 4003527; fax: +98 571 4003520.
E-mail address: m.lezgy@hsu.ac.ir

http://dx.doi.org/10.1016/j.tws.2014.07.014
0263-8231/© 2014 Elsevier Ltd. All rights reserved.

laminated composite structure with rectangular cross-section. To the author's knowledge, none of these high-order theories has been applied for the analysis of composite steel–concrete beams.

Regardless of the kinematic models of different theories, various analytical approaches have been also used in the literature to date for the analysis of composite steel–concrete beams. Based on the state-space approach, Xu and Wu [28] proposed a two-dimensional (2D) analytical solution for analyzing the simply supported composite steel–concrete beams. Foraboschi [29] presented a fully non-linear analytical exact model for analyzing composite beams under transverse bending loads. Ranzi and Bradford [30] introduced an analytical solution for the time dependent behavior of composite beams with partial interaction. Gara et al. [31] proposed short- and long-term analytical solutions for composite beams with partial interaction and shear-lag effects. Although the analytical methods lead to the solutions with high accuracy, their application is limited to the problems with simple geometries, boundary and loading conditions. In practical cases, solutions based on numerical approaches are necessary. The different numerical approaches have been used by researchers for solving different problems of composite steel–concrete beams such as static analysis [32,33], non-linear responses [34–36], time-dependent analysis [37–39], damage and failure mechanics [40], and inelastic buckling [41]. Most of these works are based on finite element approach. Indeed, the finite element method is one of the most powerful and reliable approaches for solving the complicated engineering and science problems. However, the finite element approach has some inherent drawbacks due to the use of the so-called meshing process for obtaining a discretized geometry. This process often leads to geometrical errors. Moreover, mesh generation, re-meshing and achieving the smoothness with arbitrary continuity order between the elements are the cumbersome tasks and time-consuming procedures in the traditional finite element analysis. To overcome these drawbacks, a new powerful computational method so-called isogeometric approach is recently introduced in engineering and science disciplines by Hughes et al. [42]. The conventional NURBS-based Computer Aided Design (CAD) tool is directly integrated into the finite element analysis. Indeed, isogeometric approach tries to overcome the gap between CAD and finite element analysis. Therefore, the same geometric definition of non-uniform rational B-spline basic functions is used to represent the field variable approximations of the structures as well as the geometry of structure. Isogeometric analysis based on NURBS basic functions can preserve exact geometries and enhance the accuracy of the traditional finite elements significantly. As well known, the NURBS basic functions can provide higher continuity of derivatives than that of Lagrange and Hermite interpolation functions that has been widely used in finite element formulation. In addition, the order of NURBS basic functions can be easily increased without changing the geometry or its parameterization [43]. Thanks to these features, isogeometric analysis has been used for solving different engineering problems such as fluid mechanics [44,45], structural shape optimization [46,47], damage and fracture mechanics [48], structural vibration [49], linear and non-linear elasticity and plasticity [50]. However, the isogeometric formulations have not been used for the analysis of composite steel–concrete structures yet.

In the present study, an isogeometric approach for the development of a new element based on a refined high-order theory is introduced for the analysis of composite steel–concrete beams. This theory which is based on global–local assumptions has been introduced first time by Lezgy-Nazargah et al. [25] for the analysis of laminated composite beams. The original high-order global–local theory then successfully extended to smart laminated composites by Lezgy-Nazargah and colleagues [51–53]. In this refined high-order theory, the global in-plane displacement component is described by combinations of polynomial and exponential expressions whereas the global transverse displacement component is adopted as a fourth-order polynomial. To improve the results, local terms have been added to the global expressions employing the layerwise concepts. The employed high-order global–local theory considers effects of the transverse normal stress and transverse flexibility in the analysis. Majority of the available high-order theories either do not consider the transverse flexibility or do not impose the continuity condition of the transverse normal stress at the layer interfaces. However, the transverse normal stresses and strains and the transverse flexibility which are the cause of many failure modes, have important roles in the analysis of the laminated composite structures. In the employed high-order theory, the boundary conditions of shear and normal tractions are also satisfied on the upper and lower surfaces of the beam. Besides, continuity conditions of the displacement components, transverse shear and normal stresses at the layer interfaces are satisfied. In comparison with the other available similar theories (layer-wise or 3D models available in the commercial softwares), the employed high-order theory is computationally significantly economic and has finally, only four independent generalized unknown parameters (three displacement and one rotation parameters). In this framework, the present study is focused on the extension of these last works to the isogeometric analysis of composite steel–concrete beams. The composite concrete–steel beams with various boundary conditions are treated using a written computer code whose algorithm is based on the present isogeometric formulation. The obtained numerical results exhibit a good agreement with the available published and 3D finite element (ABAQUS) results.

2. Theoretical formulations

2.1. The geometric parameters and the coordinate system

Fig. 1a shows the cross-section of a prismatic composite steel–concrete beam made of a concrete slab and a steel member. The geometric parameters of the composite beam and the chosen Cartesian coordinate system (x, y, z) are also shown in Fig. 1b. As it

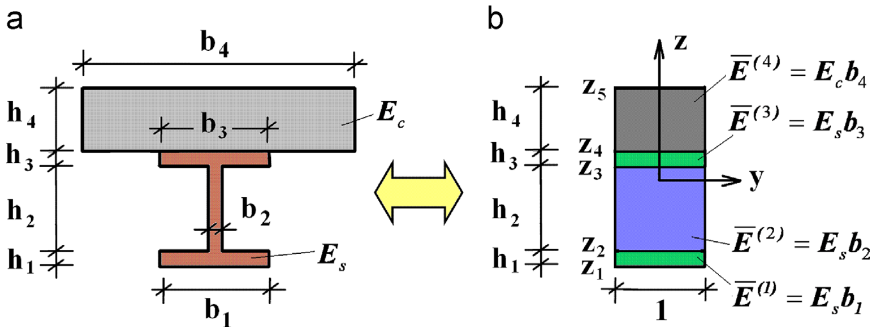


Fig. 1. (a) Typical cross-section of the composite beam and (b) equivalent laminated cross-section of the composite beam.