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A large deformation–small strain formulation for the mechanics of geometrically exact thin-walled composite beams

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ABSTRACT

This work presents a new formulation of the geometrically exact thin walled composite beam theory. The formulation assumes that the beam can undergo arbitrary kinematical changes while the strains remain small, thus compatibilizing the hypotheses of the strain measure and the constitutive law of the composite material. A key point of the formulation is the development of a pure small strain measure written solely in terms of scalar products of position and director vectors; the latter is accomplished through the obtention of a generalized small strain vector by decomposition of the deformation gradient. The resulting small strain measure is objective under rigid body motion. The finite element implementation of the proposed formulation is simpler than the finite strain theory implementation previously developed by the authors. Numerical experiments show that the present formulation is very accurate and computationally more efficient than the finite strain formulation, thus it is more convenient for most practical applications.

1. Introduction

The use of composite beams for modeling structural components is a common practice; the behavior of slender parts of modern machines such as wind turbines, satellites, cars, etc. is often predicted using the thin-walled composite beam (TWCB) approach. Good modeling practices normally imply the use of geometrically nonlinear TWCB theories, which are capable of describing not only large kinematical changes in the beam configuration but also nonlinear interactions between different components of mechanisms or multibody systems.

The thin-walled beam formulation is due to Vlasov [1]; remarkably, it has survived 50 years without drastic changes. One of the principal extensions of the theory was the inclusion of the mechanics of composite materials; several approaches that deal with the elastic behavior of TWCBs can be found in the literature and they are generally derived from Vlasov's thin walled beam theory. Although most works introduce novel aspects in their formulations, very often their hypotheses lead to geometrical or constitutive inconsistencies, or both.

The vast majority of the thin-walled beam formulations that can be found in the literature rely in the assumption of a displacement field which is introduced into a Green strain expression to obtain generalized strain measures in terms of the kinematic variables and its derivatives. Commonly the kinematic variables are taken as three displacements and three rotations, sometimes also a warping degree of freedom is used.

At least one of the following four inconsistencies can be found in almost all the works regarding TWCB, i.e. (i) the displacements field is said to describe moderate or large kinematical changes while the nonvectorial nature of the rotation variables is disregarded, (ii) a linear or second order nonlinear displacement field is assumed, but then it is introduced into an arbitrary large strain expression, (iii) some terms of the Green strain regarded as nonlinear strain measures are eliminated causing the loss of the objectivity of the resulting "linear" strain measures and (iv) the kinematic description of the formulation admits large strains while the constitutive law is only valid for small strains.

Taking, for instance, the developments by Librescu [2], it can be found that they suffer from inconsistencies (i), (ii) and (iv). Also, the works by Pi et al. [3–5] suffer from inconsistencies (i) and (iii). Analyzing their works [3,4] it can be seen that the rotation matrix is said to be second order accurate while its components are treated as vectors, thus ignoring the non-commutativity of rotations. Also, non-pure strain (higher order) terms of the Green strain measure are eliminated without testing the objectivity of the resulting strain measures. In [5] an exact rotation matrix is used, but again the elimination of non-pure strain terms cast doubt on the objectivity of the formulation; also, the rotation

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matrix is said to belong to the Special Orthogonal Group (SO3) while it is linearized as it belonged to a vector space. The theories developed by Cortínez, Piován and Machado in works [6–10] for the study of the dynamic stability, vibration, buckling and postbuckling of both open and cross section TWCBs suffer from inconsistencies (i), (ii) and (iv).

Regarding geometrically exact TWCB formulations, Saravia et al. [11,12] presented Eulerian, Total Lagrangian and Updated Lagrangian formulations using a parameterization in terms of director vectors. These formulations can describe kinematical and strain changes of arbitrary magnitude consistently; however, the constitutive law of composite laminates is only valid for small strains. A similar problem affects most of the geometrically exact formulations [13–16] developed for isotropic beams.

The mentioned works are only a few of many that present the mentioned inconsistencies. Although it can be asked if the errors that arise from these issues are actually of great influence in practical situations, the uncertainty about the limit of application of these hypotheses strongly motivates the development of a consistent approach in which a validity assessment of the theory is not needed. It is true that due to the accuracy of the modern Variational Asymptotic Methods, the use of the TWCB approach shall probably be reduced in the future. However, a vast amount of efforts are being done by researchers to improve the theory, and thus it is worth to develop a consistent large deformation–small strain formulation for thin walled composite thin-beams.

In this context, this paper presents the derivation of mathematical aspects of the finite deformation–small strain TWCB formulation. In the present approach the kinematic changes of the beam are assumed to be arbitrary, thus allowing finite rotations. The deformation gradient is written in terms of the director field; after obtaining its extended polar decomposition [17,18], a vectorial pure small strain measure is found. Finally, discrete versions of the small strain measures are found in terms of the current director and displacement fields and its derivatives; the obtained relations are remarkably simple and do not involve derivatives of the reference triads. The discrete generalized strain measures are proved to be objective under rigid body motions. The formulation is implemented in a finite element formulation; numerical results show that the proposed approach has excellent accuracy compared to the finite strain implementation previously developed by the author.

2. Kinematics

The kinematic description of the beam is extracted from the relations between two states of a beam, an undeformed reference state (denoted as $\mathcal{B}_0$) and a deformed state (denoted as $\mathcal{B}$), as it is shown in Fig. 1. Being $\boldsymbol{a}_i$ a spatial frame of reference, two

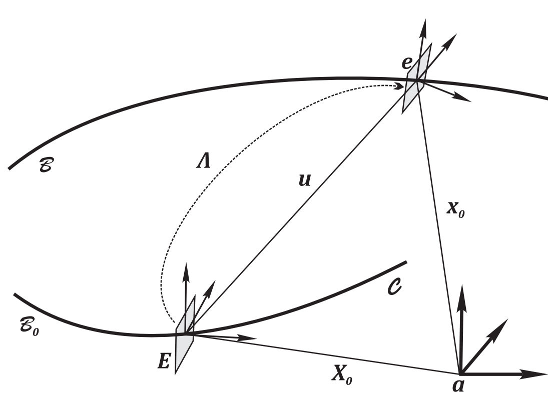

Fig. 1. 3D beam kinematics.

orthonormal frames are defined: a reference frame $\boldsymbol{E}_i$ and a current frame $\boldsymbol{e}_i$.

The displacement of a point in the deformed beam measured with respect to the undeformed reference state can be expressed in the global coordinate system $\boldsymbol{a}_i$ in terms of a vector $\boldsymbol{u}=(u_1,u_2,u_3)$.

The current frame$\boldsymbol{e}_i$ is a function of a running length coordinate along the reference line of the beam, denoted as x, and is fixed to the beam cross-section. For convenience, it is chosen the reference curve $\mathcal{C}$ to be the locus of cross-sectional inertia centroids. The origin of $\boldsymbol{e}_i$ is located on the reference line of the beam and is called *pole*. The cross-section of the beam is arbitrary and initially normal to the reference line.

The relations between the orthonormal frames are given by the linear transformations:

$$\boldsymbol{E}_i=\boldsymbol{\Lambda}_0(x)\boldsymbol{a}_i, \quad \boldsymbol{e}_i=\boldsymbol{\Lambda}(x)\boldsymbol{E}_i, \tag{1}$$

where $\boldsymbol{\Lambda}_0(x)$ and $\boldsymbol{\Lambda}(x)$ are two-point tensor fields $\in$ SO(3); the special orthogonal (Lie) group. Thus, it is satisfied that $\boldsymbol{\Lambda}_0^T\boldsymbol{\Lambda}_0= \boldsymbol{I}$, $\boldsymbol{\Lambda}^T\boldsymbol{\Lambda}=\boldsymbol{I}$. It will be considered that the beam element is straight, so $\boldsymbol{\Lambda}_0=\boldsymbol{I}$.

Recalling the relations (1), the position vectors of a point in the undeformed and deformed configurations respectively can be expressed as

$$\boldsymbol{X}(s,X_2,X_3)=\boldsymbol{X}_0(x)+\sum_{i=2}^{3}X_i\boldsymbol{E}_i, \quad \boldsymbol{x}(s,X_2,X_3,t)=\boldsymbol{x}_0(s,t)+\sum_{i=2}^{3}X_i\boldsymbol{e}_i. \tag{2}$$

where in both equations the first term stands for the position of the pole and the second term stands for the position of a point in the cross section relative to the pole. Note that x is the running length coordinate and X_2 and X_3 are cross section coordinates. At this point we note that since the present formulation is thought to be used for modeling high aspect ratio composite beams, the warping displacement is not included. As it is widely known, for such type of beams the warping effect is negligible [19].

Also, it is possible to express the displacement field as

$$\boldsymbol{u}(s,X_2,X_3,t)=\boldsymbol{x}-\boldsymbol{X}=\boldsymbol{u}_0(s,t)+(\boldsymbol{\Lambda}-\boldsymbol{I})\sum_{2}^{3}X_i\boldsymbol{E}_i, \tag{3}$$

where $\boldsymbol{u}_0$ represents the displacement of the kinematic center of reduction, i.e. the pole. The nonlinear manifold of 3D rotation transformations $\boldsymbol{\Lambda}(\boldsymbol{\theta})$ (belonging to the special orthogonal Lie Group SO(3)) is described mathematically via the exponential map [13]. The rotation tensor in component form yields

$$\boldsymbol{\Lambda}=\sum_{i,j=1}^{3}\Lambda_{ij}\boldsymbol{E}_i\otimes\boldsymbol{E}_j, \tag{4}$$

where the components Λ_{ij} of the rotation tensor can be obtained in the following form:

$$\Lambda_{ij}=\boldsymbol{E}_i\cdot\boldsymbol{\Lambda}\boldsymbol{E}_j=\boldsymbol{E}_i\cdot\boldsymbol{e}_j \tag{5}$$

then it is possible to express the rotation tensor as

$$\boldsymbol{\Lambda}=\sum_{i,j=1}^{3}(\boldsymbol{E}_i\cdot\boldsymbol{e}_j)\boldsymbol{E}_i\otimes\boldsymbol{E}_j. \tag{6}$$

Now, using the tensor product property $(\boldsymbol{a}\otimes\boldsymbol{b})\boldsymbol{c}=(\boldsymbol{c}\cdot\boldsymbol{b})\boldsymbol{a}$, it is obtained

$$\boldsymbol{\Lambda}=\sum_{i,j=1}^{3}(\boldsymbol{E}_i\otimes\boldsymbol{E}_i)\boldsymbol{e}_j\otimes\boldsymbol{E}_j=\sum_{j=1}^{3}\boldsymbol{I}\boldsymbol{e}_j\otimes\boldsymbol{E}_j, \tag{7}$$

Finally, with summation from 1 to 3 implicitly assumed, the following expression for the rotation tensor can be obtained:

$$\boldsymbol{\Lambda}=\boldsymbol{e}_j\otimes\boldsymbol{E}_j, \tag{8}$$