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Buckling analysis through a generalized beam model including section distortions

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ABSTRACT

A geometrically nonlinear beam model suitable for describing complex 3D effects due to non-uniform warpings including non-standard in-plane distortions of the cross-section is presented and applied to the buckling analysis of beams. Each section is endowed with a corotational frame where statics and kinematics are described using a refined linear elastic model which exploits a semi-analytical solution of the Cauchy continuum problem based on a FEM discretization of the cross-section. The stress field in this way is fully 3D, allowing both the exact recovery of the standard Saint Venant solution and the consideration of some additional relevant strain modes of the cross-section that are evaluated in a simple and effective way. Numerical results are presented and compared with 3D shell reference solutions obtained by using the commercial code ABAQUS.

1. Introduction

Due to their wide use in engineering practice, linear and nonlinear analyses of beam-like members represent attractive topics for researchers who aim to improve both the accuracy of the continuum models and the efficiency of the FEM solution procedures. Over the last few decades, hundreds of papers regarding beam models for linear and nonlinear analyses have been published (see [1,2] for a detailed overview of the most important proposals). Significant contributions regarding the geometric nonlinear analysis of such structures have focused on the determination of the elastic buckling load [3–5], the initial post-buckling behavior [6] and of the whole equilibrium paths [7–12] of single beams and frames.

In particular the analysis of 3D beams and frames in composite materials or thin-walled profiles requires appropriate tools, capable of accurately predicting complex 3D behaviors such as interlaminar stresses, section distortions and non-standard coupling effects [13]. Solid or shell based analyses could be very expensive in terms of computing resources for real scale problems, especially when the nonlinear behavior is considered, so the recourse to accurate 1D models capable of reproducing the essential aspects of the original solution appears to be preferable. Standard beam models which adopt a rough description of the cross section motion are inadequate for these purposes, lacking in an accurate description of complex in-plane deformation of the cross section or fully 3D states of stress.

To fill this gap, in the last few decades, the researchers' attention has been devoted to formulate refined or high order beam theories based on a decomposition of the 3D elasticity problem into a one-dimensional part defined along the beam axis and a cross-section analysis suitable for the enrichment of the beam kinematics. This is the case of the *generalized beam theory* or GBT initially proposed by Schardt [14] for the analysis of thin walled isotropic beams used in civil engineering applications. The theory has been notably improved in the last few years principally by Camotim and coauthors [12,15–17], while other recent contributions can be found in [18,19]. The fundamental idea of the GBT is that of considering the beam as an "assembly" of thin plates. Introducing other suitable simplified assumptions, the cross section analysis is then reduced to the section middle line (see also [20]). Another important contribution is the variational-asymptotic method developed essentially by Hodges and coworkers [21,22,7] in which the same goal is reached by selecting the significant terms in the elastic formulation of a 3D beam by means of an asymptotic section analysis. Other relevant contributions to the modeling of the warping effects, even if limited to the linear elastic case, can be found in [23–26].

In this paper, a new model for beams subjected to variable warpings and section distortions, undergoing large displacements and rotations, but small strains is obtained through the Implicit Corotational Method (ICM) [27]. The ICM reuses the corotational description at the continuum level by introducing a corotational reference system for each cross-section. In this system, following a

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mixed approach, the linear stress tensor is shown to be a good approximation of the Biot nonlinear one, while a quadratic approximation of the strain is easily obtained from the symmetric and the skew-symmetric parts of the displacement gradient of a parent linear solution. The two fields so defined are introduced in the Hellinger–Reissner functional to describe the beam behavior in terms of generalized static and kinematic quantities only, while change of observer algebra is used to complete the framework. The so generated nonlinear model is objective with respect to the mean rigid section motion, while its capability in describing complex phenomena depends on the parent linear model considered. Readers are referred to [28] for its first application to the Saint-Venant (SV) and the Kirchhoff solutions for beams and plates, while in [29] an extension to homogeneous and isotropic beams subjected to variable shear/torsion warping deformations is presented.

The capability of the ICM to describe complex 3D behavior depends largely on the linear solution considered. Existing linear formulations for beams only partially account for the richness of 3D continuum, introducing appropriate hypotheses for the statics and the kinematics of the body when formulating the one-dimensional model. The linear beam model in [30,31], on the contrary, is based on the solution of the full 3D elasticity problem extending the approach initially proposed by Giavotto et al. [32,33] for the analysis of helicopter rotor blades. The basic idea is that of building a coherent approximation of the beam stresses and displacements starting from a semi-analytical solution of the Cauchy continuum problem for beam-like bodies under the usual Saint Venant (SV) loading conditions also for beams of composite section. We define, for a 3D beam section, a system of homogeneous differential equilibrium equations in terms of cross section FE parameters which are solved by means of an eigenvalue problem. In particular the system matrix is characterized by a group of eigenvectors associated with null eigenvalues that define a polynomial solution along the beam axis and a group of non-null eigenvalues which represent the part of the solution associated with tip end effects and with self-balanced stresses exponentially decaying moving away from the bases. The polynomial part (*central solution*) includes the beam rigid body motion and the generalization of SV solution to generic materials (see also [34] for further details). The other modes consist in out of plane warpings as well as section distortion modes.

On the basis of the information obtained by means of the previously described *cross section analysis*, the generalized beam displacement field is approximated in terms of a rigid section motion and some other relevant strain modes (generalized warpings) independently amplified along the beam axial direction; the stress field coherently enriches that provided by the generalized SV solution through the contributions due to all the generalized warping effects considered. Differently from other proposals, the model includes in the analysis a set of strain modes of the cross-section in a coherent way thereby resulting potentially capable of accounting for the full 3D effects of the solution. It represents an alternative framework to GBT being virtually capable of accurately accounting for any kind of beam section (compact or thin-walled) and material.

Attention will be focused on the capability of the generalized warpings, obtained from the section analysis, together with the adoption of the ICM strategy, to accurately describe the bucking solution in terms of both deformation and stress fields. The analysis is carried out by considering essentially isotropic materials for which it is easy to separate the in plane and out of plane section modes and then to simply treat more complex boundary conditions. The model is, however, general and well suited also for the analysis of composites as a final example will show.

The finite element formulation is obtained from the Hellinger–Reissner functional introducing a suitable interpolation of both the stress and displacement fields. By means of a block elimination of the variables that do not require inter-element continuity, the element, at the global level, exposes only a reduced number of kinematical parameters and uses a pseudo-compatible format to perform the analysis [35,36,29]. It will be shown that very accurate results can be obtained considering few generalized warpings of the cross section also for complex buckling modes containing a localization of displacements. Numerical test results regarding both the buckling loads and modes will be compared with those furnished by shell analyses using the ABAQUS code.

2. The beam model

In this section the nonlinear beam model will be derived starting from a suitable *parent* linear formulation according to the ICM framework (see [27,29]). Some basic concepts about the section analysis and the linear beam model will be presented to introduce the notation. Interested readers are referred to [30,34] for a complete discussion about both these topics.

2.1. The use of the semi-analytic approach for the cross section analysis

Let us consider the beam as a Cauchy body referred to a fixed Cartesian frame with origin $\mathcal{O}$ and basis vectors $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\}$. Each material reference point is defined by a position vector $\boldsymbol{X} = s\boldsymbol{e}_1 + \boldsymbol{x}$, $s \equiv x_1$ being a one-dimensional abscissa along the axis line or *support* of length ℓ while $\boldsymbol{x} = x_2\boldsymbol{e}_2 + x_3\boldsymbol{e}_3$ lies on the cross section or *fiber* $\Omega[s]$.

Adopting a Voigt-like notation and omitting the dependence of the quantities when clear, the compatibility relationship between linear strains collected in $\boldsymbol{\varepsilon}[s,\boldsymbol{x}] = \{\varepsilon_{11}, \gamma_{12}, \gamma_{13}, \varepsilon_{22}, \varepsilon_{33}, \gamma_{23}\}$ and displacements $\overline{\boldsymbol{\upsilon}}[s,\boldsymbol{x}] = \{\overline{v}_1, \overline{v}_2, \overline{v}_3\}$, becomes

$$\boldsymbol{\varepsilon} = \boldsymbol{D}\overline{\boldsymbol{\upsilon}} + \boldsymbol{S}\overline{\boldsymbol{\upsilon}}_{,s}, \tag{1}$$

where operators $\boldsymbol{S}$ and $\boldsymbol{D}$ are defined as follows:

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\partial}{\partial x_2} & 0 & 0 \\ \frac{\partial}{\partial x_3} & 0 & 0 \\ 0 & \frac{\partial}{\partial x_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial x_3} \\ 0 & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_2} \end{bmatrix}$$

A proper FEM interpolation on the cross-section is introduced so to express $\overline{\boldsymbol{\upsilon}}$ in the form

$$\overline{\boldsymbol{\upsilon}}[s,\boldsymbol{x}] = \boldsymbol{N}_v[\boldsymbol{x}]\boldsymbol{q}[s], \tag{2}$$

where $\boldsymbol{N}_v$ is the shape function matrix and $\boldsymbol{q}[s]$ is the vector collecting all the n_q discrete parameters of the cross-section.

Strains in Eq. (1) can be evaluated as

$$\boldsymbol{\varepsilon} = \boldsymbol{L}_\varepsilon[\boldsymbol{x}]\boldsymbol{\psi}[s], \tag{3}$$

where

$$\boldsymbol{L}_\varepsilon[\boldsymbol{x}] = [\boldsymbol{S}\boldsymbol{N}_v \;\; \boldsymbol{D}\boldsymbol{N}_v], \quad \boldsymbol{\psi}[s] = \begin{bmatrix} \boldsymbol{q}_{,s} \\ \boldsymbol{q} \end{bmatrix} \tag{4}$$

and, from now on, a comma denotes derivative.

Letting $\boldsymbol{\sigma}[s,\boldsymbol{x}] = \{\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{22}, \sigma_{33}, \sigma_{23}\}$ be the vector collecting the stress components, we assume a linear constitutive law expressed as

$$\boldsymbol{\sigma} = \boldsymbol{C}[\boldsymbol{x}]\boldsymbol{\varepsilon}, \tag{5}$$

where the constitutive matrix $\boldsymbol{C}$ is assumed constant with s.

Substituting Eqs. (2), (3) and (5), in the virtual work equation or in the potential energy functional (see [30,34]) the equilibrium