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# Buckling of thin-walled torispherical heads in water heater tanks

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ABSTRACT

The buckling strength of a thin-walled torispherical head in a residential electric water heater tank was investigated by both experimental and finite element analyses. Three water heater tanks pressurized with water were tested, and the strains on the heads and the pressure variations were measured and recorded. Finite element analysis was used to predict the buckling of the torispherical head. The effect of the imperfection induced by the contact nonuniformity between the torispherical head and the shell on the buckling of the structure is included. Good agreement between the test and finite element results shows that finite element models used in this paper are viable to predict the buckling pressure of a thin-walled torispherical head in a water heater tank. The results also show that the contact between the bottom head and the shell has a reinforcing effect on the buckling strength of the head. The contact imperfection will produce a dent adjacent to the knuckle region when the head buckles. The buckling pressure of the head perfectly contacting with the shell is 6.88% higher than that without contacting with the shell. The obtained results provide reference for the design and manufacture of water heater tanks.



## 1. Introduction

The residential electric water heater is widely used in our daily life. The tank is the pressurized element of the water heater with a maximum operating pressure of 1.034 MPa and a maximum operating temperature of 75°C. For water heater manufacturers, an investigation of the tank design is important, because it relates to not only the safety of the water heater, but also the economics of the manufacturing process.

The water heater tank is a thin-walled vessel, which typically consists of two torispherical heads and a shell, as shown in Fig. 1. Compared with the conventional joint of a head and shell, there is a novel connection of the bottom head and shell using a single fillet lap weld where the bottom head is not only the end closure of the tank, but also the tank support, allowing the tank to be oriented in the vertical position. To assemble the bottom head and the shell, the bottom head is inserted into the shell, and then welded to the shell end. Therefore, a flare is formed at the shell end after the assembly process, and there is a contact region between the bottom head and the shell, as shown in Fig. 1. When a water heater is in operation, the bottom head is under the influence of external pressure on the convex surface. Buckling of the bottom is, therefore, a potential failure mode to be prevented in the design.

As a classical design problem, investigations of torispherical head buckling under external pressure have been performed in the past several decades. Washington et al. [1] presented an early study on the buckling of externally pressurized torispheres, where 18 pressure vessel heads having nine different geometries and made from two different steels were subjected to monotonically increasing external pressure until collapse occurred. Galletly and Kruzelecki [2] used the finite element method to investigate the buckling of shallow dished ends under external pressure. The results showed that the application of BS 5500 to the design of shallow steel torispheres subjected to external pressure can result in safety factors which are lower than the expected 1.5. Lu et al. [3] performed a systematic numerical investigation of the nonlinear elastic and elastic–plastic load-carrying behavior and imperfection sensitivity of torispherical pressure heads under uniform external pressure. Blachut [4] studied the buckling of sharp knuckle torispherical shells under external pressure and pointed out that the safety margin as used for externally pressurized hemispheres and deep torispheres was inadequate for sharp knuckle torispheres with a knuckle radius of 6% of diameter. Blachut [5] presented results of the numerical and experimental investigation into the static stability of externally pressurized hemispherical and torispherical domes fabricated with multilayered metals. Recently, Blachut [6] published a comprehensive review on the buckling of the pressure vessel components. He observed that the radius of the knuckle in the torisphere needs to be considered in the design, and torispheres with a knuckle radius of approximately equal to 6% of diameter might collapse well below the recommended design curve. In all of these examples of the open literature, the investigated torispherical head is either considered separated from

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**Nomenclature**

| | |
|---|---|
| $e$ | true strain of the steel |
| $D_{ho}$ | outside diameter of the bottom head |
| $D_i$ | inside diameter of the shell |
| $h$ | height of the bottom head |
| $L_c$ | outside circumference of the bottom head |
| $L_{im}$ | imperfection width of the contact between the shell and the bottom head in the circumferential direction of a tank |
| $P$ | pressure imposed on the bottom head |
| $P_{buckle}^{test}$ | buckling pressure from a test |
| $P_{buckle}^{AL}$ | buckling pressure from finite element analysis using the Arc-Length method |
| $P_{buckle}^{NR}$ | buckling pressure from finite element analysis using the Newton–Raphson method |
| $P_{buckle}^{FEA}$ | buckling pressure from finite element analysis |
| $r_i$ | inside knuckle radius of the bottom head |
| $R_i$ | inside crown radius of the bottom head |
| $R_{im}$ | ratio of the contact imperfection width to the outside circumference of the bottom head |
| $s$ | true stress of the steel |
| $U_{max}$ | maximum displacement of the bottom head |
| $x$ | arc length away from the bottom head end in the meridional direction |
| $\varepsilon_{max}^{test}$ | maximum measured strain on the bottom head |
| $\varepsilon_{meri}$ | meridional strain |
| $\sigma_e$ | elastic stress on the bottom head |

a shell or connected with a cylindrical shell by a butt weld. The buckling strength of the structure shown in Fig. 1 has attracted less attention. In addition, the practices of the water heater manufacturers show that the contact between the bottom head and the shell is not uniform due to the manufacture process.

In this paper, the hydrostatic test of a water heater tank was performed, and then finite element analysis was used to investigate the buckling strength of the thin-walled torispherical head in the novel connection structure of the torispherical head and the shell. The effect of imperfections induced by contact nonuniformities between the bottom head and shell on the buckling of the structure was taken into account. The obtained results can provide some reference for the water heater tank design and manufacture.

## 2. The hydrostatic test of the water heater tank

### *2.1. Test procedure*

In the hydrostatic test, three water heater tanks with the same nominal geometry were randomly selected from a large number of the products. The manufacturing plant produces nearly two million water heaters every year. To produce these products cost effectively, the tanks are produced at a fast rate, and there needs to be a high level of consistency in all aspects of the manufacturing process—steel gauge, and head, bottom, and shell dimensions. The shape measurements of 30 bottom heads were performed on a CMM (Coordinate Measuring Machine), as shown in Fig. 2. The CMM is a Zeiss Contura G2 with a VAST XT active scanning head. The measuring range in three axial directions is 700 mm, 1000 mm, and 600 mm respectively, and all axes have air bearings and ceramic guide-ways. The shape measurement results in Table 1 show that the bottom head shape is consistent for all the measured heads, and the circularity of the head flange is much less than 1% of the nominal diameter, meeting the requirements of ASME Boiler and Pressure Vessel Code Section VIII, Division 2 [7].

Strains at the concave surface of the bottom head were measured with strain gauges. Gauge resistance and gauge factor were $120 \pm 0.2\ \Omega$ and $2.08 \pm 1\%$, respectively. Strain gauges were installed on the concave surface of the bottom head in the meridional and circumferential directions. The locations of the strain gauges are illustrated in Fig. 3. In order to determine the

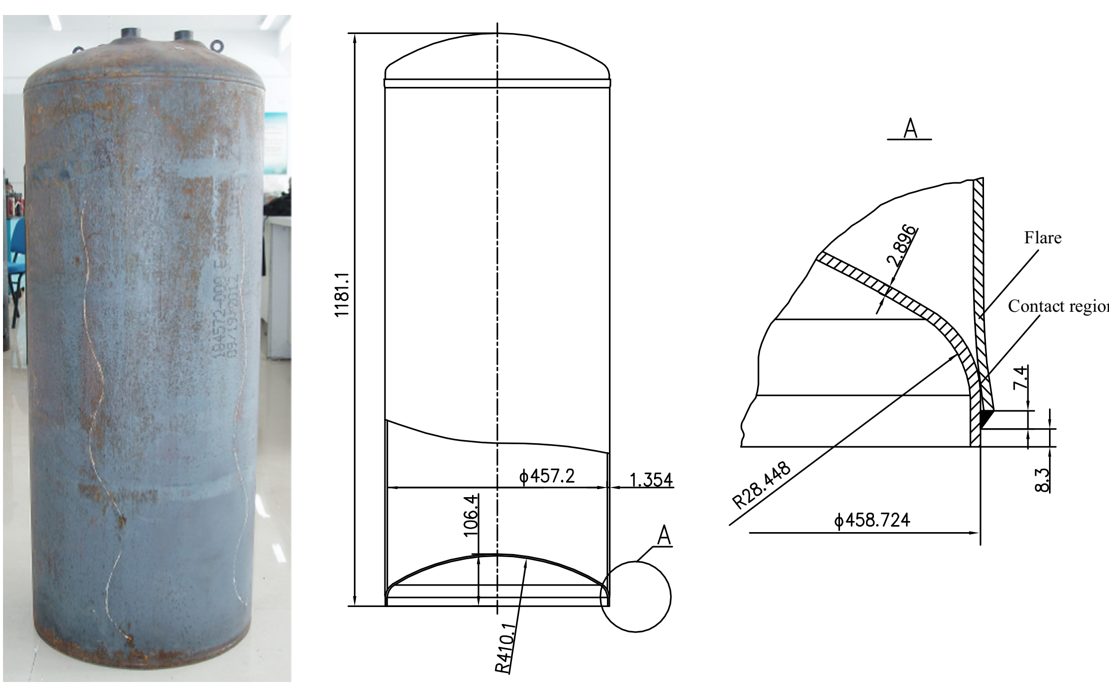


**Fig. 1.** The structure and the dimensions of the water heater tank (in mm).