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Free vibration analysis of joined conical shells: Analytical and experimental study

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ABSTRACT

Natural frequencies and mode shapes of two joined isotropic conical shells are presented in this study. The joined conical shells can be considered as the general case for joined cylindrical–conical shells, joined cylinder–plates or cone–plates, conical and cylindrical shells with stepped thicknesses and also annular plates. Governing equations are obtained using thin-walled shallow shell theory of Donnell and Hamilton's principle. The continuity conditions at the joining section of the cones are appropriate expressions among stress resultants and deformations. The equations are solved assuming trigonometric response in circumferential and series solution in meridional directions and all combinations of boundary conditions can be assumed in this method. The results are compared and validated with the available results in other investigations and also modal testing. The effects of semi-vertex angles and meridional lengths on the natural frequency and circumferential wave number of joined shells are investigated.

1. Introduction

The joined shells of revolution have many applications in various branches of engineering such as mechanical, aeronautical, marine, civil and power engineering. These structures are made of large and thin shell elements; therefore their vibration characteristic is of prime importance.

There are a few publications on the vibration analysis of joined conical–cylindrical shells. The classic bending theory was used to examine rotationally symmetric shells [1,2]. The analytical and experimental work was performed by Lashkari and Weingarten [3] using finite element method to determine the natural frequencies and mode shapes of coupled conical–cylindrical shells. A transfer matrix approach was used by Irie et al. [4] to solve the free vibration of coupled cylindrical–conical shells. Efraim and Eisenberger [5] applied a power series solution to calculate the natural frequencies of segmented axisymmetric shells using the theory of Reissner–Naghdi. Patel et al. [6] presented results for laminated composite joined conical–cylindrical shells using a finite element method. A joined complete cone–cylinder was investigated analytically by Tavakoli and Singh [7] using the state space method. The joined complete cone–cylinder was also investigated using the FEM by Özakça and Hinton [8] with a 305 DOF cubic four-noded C^0 Mindlin–Reissner finite element model. Benjeddou [9] analyzed the complete cone–cylinder combination by a local–global B-spline finite element method formulation using two super elements with 10 COREBS_M type finite element based on a classical thin shell theory of Kirchhoff–Love type. El Damatty et al. [10] performed experimental and numerical investigation to assess the behavior of the combined conical–cylindrical shells. The tested specimen is modeled numerically using a finite element program based upon a thick shell theory. Sivadas and Ganesan [11] have analyzed cylinder–cone, cylinder–plate and stiffened shells for their free vibration characteristics using a high-order semi-analytical finite element solution. Kamat et al. [12] studied the dynamic instability of a joined conical-cylindrical shell subjected to periodic in-plane load using C^0 two-noded shear deformable shell element. Lee et al. [13] studied the free vibration characteristics of the joined spherical–cylindrical shell with various boundary conditions using Flügge shell theory and modal testing.

The influences of small curvatures on the modal characteristics of a plate–shell combinations were studied by Huang [14]. Recently, Caresta and Kessissoglou [15] analyzed the free vibrations of joined truncated conical–cylindrical shells. The displacements of the conical sections were solved using a power series solution, while a wave solution was used to describe the displacements of the cylindrical sections. Both Donnell–Mushtari and Flügge equations of motion were used. To validate their analytical method, a computational finite element model was developed using Patran/Nastran and quadratic eight node elements were used for the 2-D thin shell elements. They also compared their

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results with other papers. Kang [16] studied the free vibration frequencies of joined thick conical–cylindrical shells of revolution with variable thickness using three-dimensional (3-D) formulation and Ritz method for eigenvalue solution. Qu et al. [17] studied the dynamic response of ring-stiffened conical–cylindrical shells subjected to different boundary conditions using Reissner–Naghdi thin shell theory and variational method. The method involves partitioning of the stiffened shell into appropriate shell segments in order to accommodate the computing requirement of high-order vibration modes and responses. However, the vibrational behavior of joined conical shells seems to need more investigations.

The joined conical shells can be considered as the general case for joined cylindrical–conical shells [4,6,12,13,16], joined cylinder–plates [18,19] or cone–plates [20], conical and cylindrical shells with stepped thicknesses [21] and also annular plates [22]. Therefore, the study of joined conical shells is of the main importance.

In this study, the authors investigate the natural frequencies and mode shapes of two joined isotropic conical shells. The governing equations are obtained using thin-walled shallow shell theory of Donnell-type and Hamilton's principle. The continuity conditions at the joining section of the cones are appropriate expressions among stress resultants and deformations. The equations are solved by assuming trigonometric response in circumferential and series solution in meridional direction and all combinations of boundary conditions can be assumed in this method. The results are compared and validated with the available results in other papers and also modal testing. The effects of semi-vertex angles and meridional lengths on the natural frequency and circumferential wave number of joined shells are investigated.

2. Governing equations for joined conical shells

2.1. Displacements and strains

Consider a set of two joined isotropic conical shells with (x,θ,z) coordinates, shown in Fig. 1, where x is the coordinate along the cones' generators with the origin placed at the middle of the generators, θ is the circumferential coordinate, and z is the coordinate normal to the cones' surfaces. R_1, R_2 and R_3 are the radii of the system of cones at its first, middle and end, respectively. The angles α_1 and α_2 are the semi-vertex angles of cones and L_1 and L_2 are the cone lengths along the generators. The thicknesses of cones are h_1 and h_2. The displacements of the shell's middle surface are denoted by u, v and w along x, θ and z directions, respectively.

The Kirchhoff hypothesis requires the displacement field (u, v, w) to be such that

$$
\begin{aligned}
u(x,\theta,z,t) &= u_0(x,\theta,t)+z\,\beta_x(x,\theta,t)\\
v(x,\theta,z,t) &= v_0(x,\theta,t)+z\,\beta_\theta(x,\theta,t)\\
w(x,\theta,z,t) &= w_0(x,\theta,t)
\end{aligned}
\tag{1}
$$

where (u_0, v_0, w_0) and (β_s, β_θ) represent midplane displacements and rotation of tangents along the x and θ, respectively.

Using Donnell thin shell theory, the strains and curvature changes in the middle surface of each cone can be written as follows [23]:

$$
\begin{Bmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix}
\tag{2}
$$

$$
\{\varepsilon\} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{1}{R(x)}\left(\frac{\partial v}{\partial \theta}+u\ \sin\ \alpha+w\ \cos\ \alpha\right) \\ \frac{1}{R(x)}\frac{\partial u}{\partial \theta}-\frac{1}{R(x)}v\ \sin\ \alpha+\frac{\partial v}{\partial x} \end{Bmatrix}
\tag{3}
$$

$$
\{\kappa\} = \begin{Bmatrix} \kappa_x \\ \kappa_\theta \\ \kappa_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{1}{R(x)}\left(\beta_x\ \sin\ \alpha+\frac{\partial \beta_\theta}{\partial \theta}\right) \\ \frac{1}{R(x)}\frac{\partial \beta_x}{\partial \theta}+\frac{\partial \beta_\theta}{\partial x}-\frac{1}{R(x)}\beta_\theta\ \sin\ \alpha \end{Bmatrix}
\tag{4}
$$

where $R(x)$ is the radius of the cone at any point along its length and may be expressed as follows:

$$
R(x) = R_0 + x\ \sin\ \alpha
\tag{5}
$$

and

$$
\beta_x = -\frac{\partial w}{\partial x} \quad , \quad \beta_\theta = -\frac{1}{R}\frac{\partial w}{\partial \theta}
\tag{6}
$$

The parameters $(\varepsilon_x, \varepsilon_\theta, \gamma_{x\theta})$ are the membrane strains, and $(\kappa_x, \kappa_\theta, \kappa_{x\theta})$ are the flexural (bending) strains, known as the curvatures.

2.2. Constitutive relations

The relationship between the stress and strain components in isotropic materials is as follows:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \gamma_{x\theta} \end{Bmatrix}
\tag{7}
$$

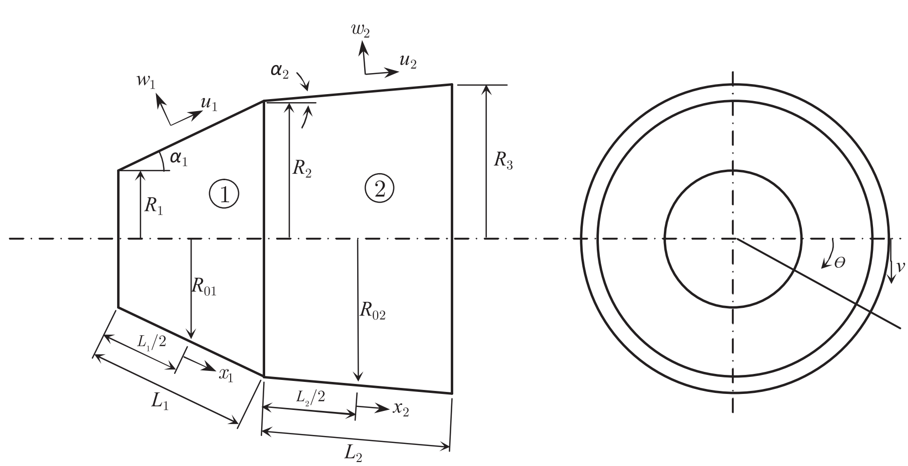

Fig. 1. Geometry of two joined conical shells.