Thin-Walled Structures 85 (2014) 377–387

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Bending collapse theory of thin-walled twelve right-angle section beams

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ARTICLE INFO

Article history:
Received 6 June 2014
Received in revised form
22 August 2014
Accepted 18 September 2014
Available online 11 October 2014

Keywords:
Theoretical bending collapse mechanism
Twelve right-angle section beam
Theoretical calculation
Quick design

ABSTRACT

The present paper focuses on the bending collapse behavior of the twelve right-angle section (TTRS) beams. This paper presents the theoretical bending collapse mechanism of the TTRS beams around two axes based on the kinematic approach, and derives the expressions of the bending moments. The accuracy of the theoretical calculations are validated respectively by performing 36 groups finite element simulations including three kinds of materials, three different section dimensions and four different thicknesses. The results show that for $17.5 < c/h < 48.6$, the theoretical bending collapse mechanism of the TTRS beams presented in this paper can describe the collapse process accurately, and the moment–rotation curves calculated by theory show a good consistency with simulation results.

1. Introduction

Thin-walled beams are widely used as key safety components in automobiles, trains, aircraft, etc. During a crash event, crushing and bending are two basic deformation modes of thin-walled beams. A thin-walled beam is more likely to bend when subjected to external impact load for the reason that bending is unstable and absorbs less energy compared to crushing. Therefore, it is important to study the bending behavior of thin-walled beams.

The bending behavior of a thin-walled beam is largely dependent on its section shape, dimension and material. The design of the bending behavior is usually conducted by using finite element simulation and experiment. However, FE simulation is highly time consuming due to the frequent change of design in the early phase. In contrast, by using the kinematic or macro element approach, i.e. modeling the simplified theoretical model and analyzing the collapse and energy dissipation mechanism, the bending moment expression with section size and thickness of a thin-walled beam can be obtained, then the bending behavior can be predicted quickly.

Since the 1980s, some researchers have begun the study of the bending behavior of thin-walled rectangular section beams. Kecman [1] used the concept of “effective” flange, derived the maximum bending moment and proposed a theoretical model for thin-walled rectangular section beams. The bending moments computed from Kecman's theory showed a good agreement with test results. Wierzbicki [2] proposed a simplified theoretical expression that expressed the relationship between the bending moment and the rotation angle in a small angle range. Kim and Reid [3] improved the model of Wierzbicki and introduced the concept of toroidal surface, and the unknown parameters could be obtained theoretically.

To reach the higher requirements of crashworthiness and lightweight, thin-walled multi-right-angle section beams have been widely discussed in recent years. Studies have pointed out that the load capacity of a thin-walled multi-right-angle section beam is higher than a rectangular section beam due to the increased number of right angles [4,5]. At present, the thin-walled twelve right-angle section (TTRS) beams have been used in composite material autobodies as longitudinal beams since the cross section has a good symmetry which looks like a “dumbbell” [6–8].

Chen G [9] performed a substantial number of crushing and bending finite element simulations for the TTRS beams including different kinds of materials and section dimensions. The simulation results indicate that the specific energy absorption (SEA) of the TTRS beam is about 3 times of the rectangular section beam in crushing condition when the two beams have the same material, thickness, perimeter of section and width-height ratio; and this value is about 1.4 to 1.9 in bending condition when the bending angle is 25°. This means that crashworthiness and lightweight can be obtained at the same time by using the TTRS beam instead of the rectangular section beam. And with the development of forming manufacturing craft, the TTRS beams will be widely used. This paper chooses the TTRS beams as the study instance, presents the simplified theoretical models and collapse mechanism of the

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http://dx.doi.org/10.1016/j.tws.2014.09.016
0263-8231/

TTRS beams bending around two axes based on the bending theory of rectangular section beams, and derives the expressions of the bending moments. The analysis can help in a quick design of the section shape, dimension and material, and provide a prediction of the bending behavior and lightweight effect for the TTRS beams in the early design phase.

2. Bending collapsed theory of thin-walled twelve right-angle section beams around the *y* axis

The section dimension of the TTRS beam is shown in Fig. 1. The width in y direction is b_y, the concave width is b_a, and the length in z direction is b_1, b_2, b_3, and $b_z = b_1 + b_2 + b_3$. The thickness of the wall is h, and the material yield stress is σ_y, the limit stress is σ_u. According to the empirical equation, the equivalent material flow stress is [10]:

$$\sigma_0 = 2.23^n \frac{\sigma_u}{n+1}\left(\frac{2}{n+2}\right)^{2/3}\left(\frac{h}{b}\right)^{4n/9} \tag{1}$$

where n is the material hardening factor. The average fully plastic moment of a unit width of the wall is:

$$M_0 = \frac{\sigma_0 h^2}{4} \tag{2}$$

The bending theory of rectangular section beams can be used in the research of the bending theory of the TTRS beams since their sections are all composed of right angles. According to Kecman's theory, the bending theory of the TTRS beams derived in this paper applies to the first two phases of the bending process, namely the maximum bending angle is 25°–35°. By making the twelve right-angle section equivalent to the rectangular section, the maximum bending moment around the y axis M_{maxy} of the TTRS beam can be calculated by using the expressions of the maximum bending moment of the rectangular section beam as shown in Eqs. (3a)–(3c), where σ_{cr} is the critical buckling stress, σ_y is the material yield stress, a and b are replaced by $(b_y + 2b_a)$ and $(b_1 + b_2 + b_3)$, respectively.

If $\sigma_{cr} < \sigma_y$:

$$M_{\max} = \sigma_y h b^2 \frac{2a + b + a(0.7(\sigma_{cr}/\sigma_y) + 0.3)(3(a/b) + 2)}{3(a+b)} \tag{3a}$$

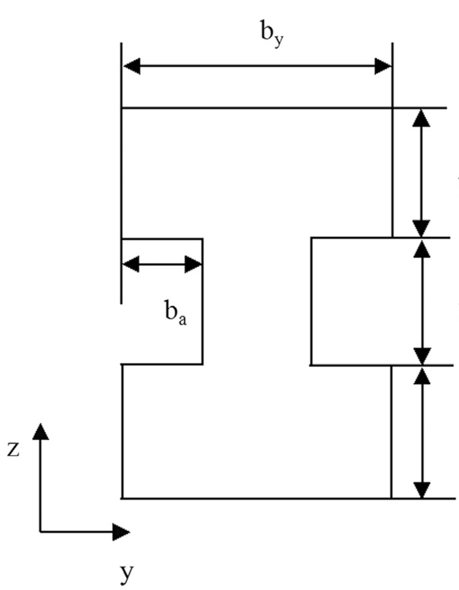

Fig. 1. Section dimension of the TTRS beam.

If $\sigma_y < 2\sigma_{cr}$:

$$M_{\max} = M_p = \sigma_y h\left[a(b-h) + 0.5(b-2h)^2\right] \tag{3b}$$

If $\sigma_y < \sigma_{cr} < 2\sigma_y$:

$$M'_p = \sigma_y h b\left(a + \frac{b}{3}\right) \quad M_{\max} = M'_p + (M_p - M'_p)\frac{\sigma_{cr} - \sigma_y}{2\sigma_y} \tag{3c}$$

Referring to the bending theory of rectangular section beams and the FE simulation result (Fig. 2), and considering the stretching of the walls, the theoretical bending collapse mechanism of the TTRS beam around the y axis is simplified as shown in Fig. 3. The energy dissipation mechanism consists of stationary hinge lines, rolling hinge lines and stretching of the walls. In Fig. 3, the structure is symmetrical about the z axis, the hinge length in z direction is $2H$, the rotation angle is o ($o = 2\rho$), and the positions of the key points are shown in Fig. 4. The expressions of the energy absorbed along one side of the lines are derived as follows, and the energy absorbed by the other symmetrical structure is added when calculating the total energy. In order to calculate the energy dissipation conveniently, the collapsed structure is divided into three parts, comprising the upper part, the middle part and the

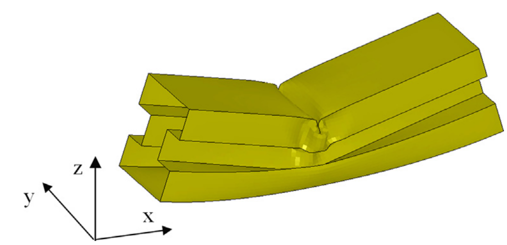

Fig. 2. Bending collapse simulation of the TTRS beam around the *y* axis.

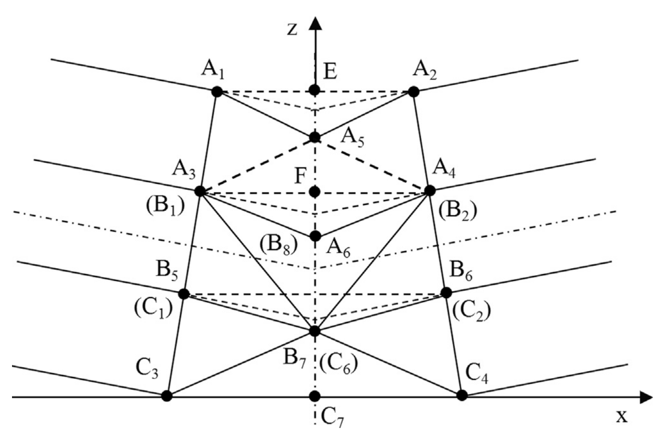

Fig. 3. Theoretical bending collapse mechanism of the TTRS beam around the *y* axis.

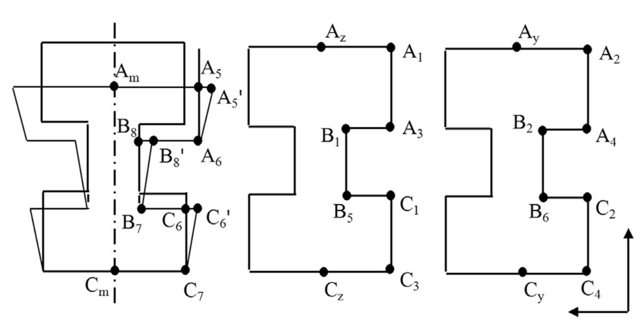

Fig. 4. Positions of key points.