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Local–global mode interaction in stringer-stiffened plates

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ABSTRACT

A recently developed nonlinear analytical model for axially loaded thin-walled stringer-stiffened plates based on variational principles is extended to include local buckling of the main plate. Interaction between the weakly stable global buckling mode and the strongly stable local buckling mode is highlighted. Highly unstable post-buckling behaviour and a progressively changing wavelength in the local buckling mode profile are observed under increasing compressive deformation. The analytical model is compared against both physical experiments from the literature and finite element analysis conducted in the commercial code Abaqus; excellent agreement is found both in terms of the mechanical response and the predicted deflections.

1. Introduction

Thin-walled stringer-stiffened plates under axial compression are well known to be vulnerable to buckling where local and global modes interact nonlinearly [1–4]. However, since stiffened plates are highly mass-efficient structural components, their application is ubiquitous in long-span bridge decks [5], ships and offshore structures [6], and aerospace structures [7,8]. Hence, understanding the behaviour of these components represents a structural problem of enormous practical significance [9–11]. Other significant structural components such as sandwich struts [12], built-up columns [13], corrugated plates [14] and other thin-walled components [15–18] are also well-known to suffer from the instabilities arising from the interaction of global and local buckling modes.

In the authors' recent work [19], the aforementioned problem was studied using an analytical approach by considering that interactive buckling was wholly confined to the stringer (or stiffener) only. So-called "cellular buckling" [20,21,18,22] or "snaking" [23–25] was captured, where snap-backs in the response, showing sequential destabilization and restabilization and a progressive spreading of the initial localized buckling mode, were revealed. The results showed reasonably good comparisons with a finite element (FE) model formulated in the commercial code Abaqus [26]. The current work extends the previous model such that the interaction between global Euler buckling and the local buckling of the main plate, as well as the stiffener, is accounted. A system of nonlinear ordinary differential equations subject to integral constraints is derived using variational principles and is subsequently solved using the numerical continuation package Auto-07p [27]. The relative rigidity of the main plate–stiffener joint is adjusted by means of a rotational spring, increasing the stiffness of which results in the erosion of the snap-backs that signify cellular buckling. However, the changing local buckling wavelength is still observed, although the effect is not quite so marked as compared with the case where the joint is assumed to be pinned [19]. A finite element model is also developed using the commercial code Abaqus for validation purposes. Moreover, given that local buckling of the main plate is included alongside the buckling of the stiffener in the current model, which is often observed in experiments, the present results are also compared with a couple of physical test results from the literature [2]. The comparisons turn out to be excellent both in terms of the mechanical response and the physical post-buckling profiles.

2. Analytical model

Consider a thin-walled simply supported plated panel that has uniformly spaced stiffeners above and below the main plate, as shown in Fig. 1, with panel length L and the spacing between the stiffeners being b. It is made from a linear elastic, homogeneous and isotropic material with Young's modulus E, Poisson's ratio ν and shear modulus $G = E/[2(1+\nu)]$. If the panel is much wider than long, *i.e.* $L \ll n_s b$, where n_s is the number of stiffeners in the panel, the critical buckling behaviour of the panel would be strut-like with a half-sine wave eigenmode along the length. Moreover, this would allow a portion of the panel that is representative of its

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entirety to be isolated as a strut as depicted in Fig. 1, since the transverse bending curvature of the panel during initial post-buckling would be relatively small.

Therefore, the current paper presents an analytical model of a representative portion of an axially compressed stiffened panel, which simplifies to a simply supported strut with geometric properties defined in Fig. 2. The strut has length L and comprises a main plate (or skin) of width b and thickness t_p with two attached longitudinal stiffeners of heights h_1 and h_2 with thickness t_s, as shown in Fig. 2(b). The axial load P is applied at the centroid of the whole cross-section denoted as the distance $\bar{y}$ from the centre line of the plate. The rigidity of the connection between the main plate and stiffeners is modelled with a rotational spring of stiffness c_p, as shown in Fig. 2(c). If $c_p=0$, a pinned joint is modelled, but if c_p is large, the joint is considered to be completely fixed or rigid. Note that the rotational spring with stiffness c_p only stores strain energy by local bending of the stiffener or the main plate at the joint coordinates $(x=0, y=-\bar{y})$ and not by rigid body rotation of the entire joint in a twisting action.

2.1. Modal descriptions

To model interactive buckling analytically, it has been demonstrated that shear strains need to be included [28,29] and for thin-walled metallic elements Timoshenko beam theory has been shown to be sufficiently accurate [21,18]. To model the global buckling mode, two degrees of freedom, known as "sway" and "tilt" in the literature [30], are used. The sway mode is represented by the displacement W of the plane sections that are under global

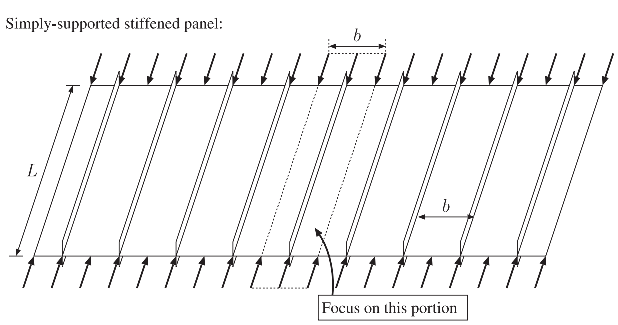

Fig. 1. An axially compressed simply supported stiffened panel of length L and evenly spaced stiffeners separated by a distance b.

flexure and the tilt mode is represented by the corresponding angle of inclination θ of the plane sections, as shown in Fig. 2(d). From linear theory, it can be shown that $W(z)$ and $\theta(z)$ may be represented by the following expressions [30]:

$$W(z) = -q_s L \sin\frac{\pi z}{L}, \quad \theta(z) = q_t \pi \cos\frac{\pi z}{L}, \tag{1}$$

where the quantities q_s and q_t are the generalized coordinates of the sway and tilt components respectively. The corresponding shear strain γ_{yz} during bending is given by the following expression:

$$\gamma_{yz} = \frac{dW}{dz} + \theta = -(q_s - q_t)\pi \cos\frac{\pi z}{L}. \tag{2}$$

In the current model, only geometries are chosen where global buckling about the x-axis is critical.

The kinematics of the local buckling modes for the stiffener and the plate are modelled with appropriate boundary conditions. A linear distribution in y for the local in-plane displacement $u(y,z)$ is assumed due to Timoshenko beam theory:

$$u(y,z) = Y(y)u(z), \tag{3}$$

where $Y(y) = (y+\bar{y})/h_1$, as depicted in Fig. 3(a).

Formulating the assumed deflected shape, however, for out-of-plane displacements of the stiffener $w(y,z)$ and the main plate $w_p(x,z)$, see Fig. 3(b), the stiffness of the rotational spring c_p, depicted in Fig. 2(c), is considered. The role of the spring is to resist the rotational distortion from the relative bending of the main plate and the stiffener with respect to the original rigid body configuration. The shape of the local buckling mode along the depth of the stiffener and along the width of the main plate can be therefore estimated, using the Rayleigh–Ritz method [31], by a nonlinear function that is a summation of both polynomial and trigonometric terms. The general form of these approximations can be expressed by the following relationships:

$$w(y,z) = f(y)w(z), \quad w_p(x,z) = g(x)w_p(z), \tag{4}$$

where

$$f(y) = B_0 + B_1 Y + B_2 Y^2 + B_3 Y^3 + B_4 \sin(\pi Y),$$
$$g(x) = C_0 + C_1 X + (-1)^i C_2 X^2 + C_3 X^3 + C_4 \sin(\pi X), \tag{5}$$

and $X(x) = x/b$. Moreover, for $i=1$, the range $x=[0, b/2]$ and for $i=2$, the range $x=[-b/2, 0]$. For $f(y)$, the constant coefficients B_0, B_1, B_2, B_3 and B_4 are determined by applying appropriate boundary conditions for the stiffener. At the junction between the stiffener

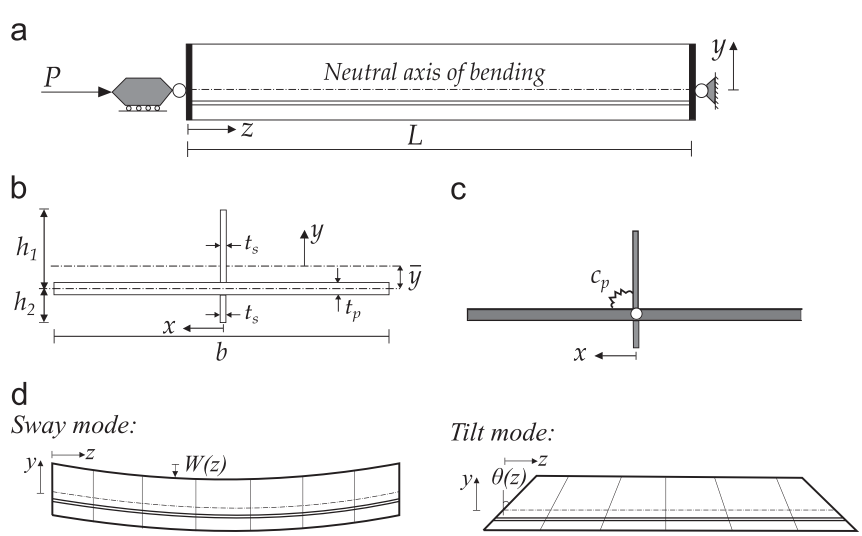

Fig. 2. (a) Elevation of the representative portion of the stiffened plate modelled as strut of length L that is compressed axially by a force P. (b) Strut cross-section geometry. (c) Modelling the joint rigidity of the main plate–stiffener connection with a rotational spring of stiffness c_p. (d) Sway and tilt components of the global buckling mode.