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An analytical method for the buckling analysis of cylindrical shells with non-axisymmetric thickness variations under external pressure

Licai Yang [a,*], Ying Luo [b], Tian Qiu [b], Min Yang [b], Gaobin Zhou [b], Guofu Xie [b]

[a] National Key Laboratory of Reactor System Design Technology, Nuclear Power Institute of China, 25 South Section 3, 2nd Ring Road, Chengdu, Sichuang 610041, People's Republic of China
[b] National Key Laboratory of Reactor System Design Technology, Nuclear Power Institute of China, 25 South Section 3, 2nd Ring Road, Chengdu, Sichuang 610041, People's Republic of China

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ABSTRACT

This article presents an analytical method for the buckling analysis of laterally pressured cylindrical shells with non-axisymmetric thickness variations. The previous results for thickness variations under external pressure are reviewed firstly. Then, a general analytical method that combines the perturbation method and Fourier series expansion is developed to derive buckling load formulas, which is in terms of thickness variation parameter up to arbitrary order. A classical non-axisymmetric thickness variation is discussed in detail by the presented analytical method. When non-axisymmetric modal thickness variation becomes axisymmetric, the buckling loads degenerate to the known results. Furthermore, the influence of circumferential modal thickness variation with mode corresponding to twice the circumferential buckling mode on the buckling of laterally pressured cylindrical shells is analytically investigated and the results show a great agreement with previous numerical ones by Gusic et al. Thus we confirm the presented method. In addition to theoretical analysis, calculations and comparisons are also performed. The general analytical method presented in the article can be utilized to determine the buckling loads of shells with general thickness variations.

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1. Introduction

Cylindrical shells are widely used in actual engineering and a vast literature has been devoted to the buckling problems of them during the past several decades. However, most of these studies were carried out on the thin shells with constant thickness. In fact, the cylindrical shells in actual project always exhibit a variety of thickness variations such as stepwise, linear, and local, due to the design [1,2], fabrication procedure (composite shells, rolled steel sheets, thickness tolerance) and corrosion in the aggressive environments (off-shore structures, submarines, pressure vessels in chemical industry). Therefore, stability evaluation of these thin walled shells with variable thickness is rather important and meaningful. In recent years, buckling of cylindrical shells with thickness variations has also attracted some attentions. Koiter et al. [3] and Elishakoff et al. [4] pioneered the investigation of the influence of axisymmetric thickness variation on the buckling of cylindrical shells under axial compression by the hybrid perturbation-Galekin method. In their paper, the thickness variation is simple and is taken to be axisymmetric and periodical with mode corresponding to twice the axial buckling mode. The results indicated that small thickness variation could provoke buckling load reduction remarkably. Therefore, they appealed to researchers that more attentions should be paid to this subject. Further investigation on the axial buckling of composite cylindrical shells with same thickness variation is performed by Li et al. [5] using the same method, and it showed that composite shells were as sensitive to the thickness variation as metallic shells. For modal circumferential thickness variation, Gusic et al. [6] studied the influence of harmonic thickness variation in the circumferential direction on the buckling of cylindrical shells under external pressure by means of finite element bifurcation analysis. Combescure and Gusic [7] also analyzed the effect of both circumferential harmonic thickness variation and geometric variation on thin cylindrical shell under external pressure by using the COMI axisymmetric shell element. Papadopoulos and Papadrakakis [8,9] developed a methodology for the stochastic finite element analysis of thin shells with combined random initial geometric, material and thickness variation. Aghajari et al. [10] experimentally and numerically investigated the buckling of thin cylindrical shells with stepwise variable wall thickness under uniform external pressure. Recently, Nguyen

* Corresponding author.
E-mail address: yanglicai@zju.edu.cn (L. Yang).

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et al. [11] considered the buckling problem of cylindrical shells with axisymmetric thickness variation in the shape of the axial buckling mode under the external pressure by employing the hybrid perturbation-Galekin method. The results showed that thin cylindrical shells subjected to external pressure were more sensitive to thickness variation than ones under axial compression. By assuming the stepwise variable wall thickness of thin shell to be uniform in the specific load condition, Yang et al. [12] performed an elastic-plastic instability analysis for large oil storage tanks and obtained an analytical formula.

The previous analytical studies [3–5,11] on the buckling of thin-walled cylindrical shells with thickness variation treated simple axisymmetric modal thickness variation by employing the hybrid perturbation-Galekin method. However, this method did not give an unified analysis for axisymmetric thickness variations, and is also very difficultly used to solve buckling load even for this simple axisymmetric modal thickness variation [11], because the solution satisfying boundary conditions of compatibility equation needs to be solved exactly. Furthermore, the cylindrical shells in actual project always exhibit a variety of thickness variations. In recent works, by combining the separation of variables and perturbation method, Chen et al. [13] have extended the previous analytical investigations and presented a unified method for the buckling of axially compressed cylinders with arbitrary axisymmetric thickness variations. They analyzed three axisymmetric thickness variations as examples and confirmed the presented results. Following the same method, Yang et al. [14] also studied the buckling of thin shells under external pressure with general axisymmetric thickness variations. The buckling loads degenerated to those of Nguyen et al. [11] when thickness variation is an axisymmetric modal one.

However, all the present analytical methods can only treat axisymmetric thickness variations. Because basic equations will be simplified as ordinary differential equations by separation of variables, the solution is relatively simple when thickness variations are axisymmetric [3–5,11,13,14]. For non-axisymmetric thickness variations, the solution will become rather difficult due to the complicated fourth-order partial differential equations with variable coefficients. Hitherto, no general method has been developed for general thickness variations. In this article, thickness variations can vary arbitrarily in both axial and circumferential directions. A general method that combines the perturbation method and Fourier series expansion will be developed to obtain buckling load for cylindrical shells with non-axisymmetric thickness variations under external pressure. A classical non-axisymmetric thickness variation example is discussed in detail and the obtained results will degenerate to the previous ones when thickness variations become axisymmetric. Furthermore, circumferential modal thickness variation with mode corresponding to twice the circumferential buckling mode will also be investigated by the presented analytical method and the results will be compared with ones by FE bifurcation analysis [6]. This study is a generalization and extension of the former investigations.

2. The previous studies on thickness variations under external pressure

2.1. Axisymmetric modal thickness variation

In the previous work, Nguyen et al. [11] analyzed the following thickness variation by applying the hybrid perturbation-Galekin method.

$$t(x) = t_0\left(1-\varepsilon\ \cos\ \frac{\pi x}{L}\right) \tag{1}$$

where $t(x)$ denotes the actual thickness of the cylindrical shell, and t_0 is its nominal thickness; $\varepsilon (0 \le \varepsilon < 1)$ is non-dimensional thickness non-uniformity parameter; L is the length of the shell; X denotes the axial coordinate (x varies from $-L/2$ to $L/2$); $\cos\ \pi x/L$ is the first-order axial buckling mode of the cylindrical shell.

When the shell is infinite long, Nguyen et al. presented the following buckling load reduction factor λ, with the help of Matlab.

$$\lambda = \frac{q}{q_0} = 1-\frac{8}{\pi}\varepsilon \tag{2}$$

where q denotes buckling load of the cylindrical shell with thickness variation of Eq. (1); q_0 is the buckling load of the cylindrical shell with the uniform thickness t_0.

From Eq. (2), it can be seen that the effect of thickness variation on the buckling of a laterally pressured cylindrical shell is quite obvious. For example, the buckling load of the infinite long cylindrical shell is reduced by 25.5% when $\varepsilon = 0.1$.

2.2. Parabolic thickness variation

Recently, Yang et al. [14] discussed arbitrary axisymmetric thickness variations by combining the separation of variables and perturbation method. They also considered thickness variation of Eq. (1) and obtained same results of Eq. (2). Furthermore, they also studied the following thickness variation.

$$t(x) = t_0\left[1-(\frac{2x}{L})^2\varepsilon\right] \tag{3}$$

For the thickness variation of Eq. (3), the obtained buckling load reduction factor can be written as:

$$\lambda = 1-\frac{\begin{array}{c}\dfrac{12(1-v^2)(\pi^2+24)}{Rt_0^2(\pi^2+m^2L^2/R^2)^2}-\dfrac{24(1-v^2)\pi^2}{Rt_0^2(\pi^2+m^2L^2/R^2)^4}[(\frac{1}{3}-\frac{2}{\pi^2})(\pi^2+m^2L^2/R^2)^2\\ -4(\pi^2+m^2L^2/R^2)]+(1-\frac{6}{\pi^2})[R(\pi^2/L^2+m^2/R^2)^2-\frac{m^2}{R^3}]+24(1-v)m^2/L^2R\end{array}}{R(\pi^2/L^2+m^2/R^2)^2+\dfrac{12(1-v^2)\pi^4}{Rt_0^2(\pi^2+m^2L^2/R^2)^2}-\dfrac{m^2}{R^3}}\varepsilon+O(\varepsilon^2) \tag{4}$$

where ν is Poisson's ratio; R is the radius of the cylindrical shell; and m represents the circumferential buckling mode of the perfect cylinder.

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