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A closed-form solution for elastic buckling of thin-walled unstiffened circular cylinders in pure flexure

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ABSTRACT

To date, despite the significant development in the field of structural mechanics, there still remains a paradox in the solutions available for a classical shell buckling problem. The difference in strength between a cylindrical shell under uniform axial compression and that under pure bending is not quite well investigated. This lack of research is reflected in the wide variations in the elastic bending strength and the slenderness limits given in current international design standards. The discrepancies in the available classical solutions and hence the design rules have initiated the current research. The main aim of this paper is to present a closed-form solution for the elastic buckling strength of unstiffened circular cylinders under pure bending using a new simplified energy approach employing the well-known Ritz method. Two types of analyses are presented for cylinders with large ($D/t > 200$) and medium ($100 < D/t < 200$) diameter-to-thickness ratios. A unique testing rig was used to experimentally verify the new theory using a Moiré fringe film. The theoretical results are compared against the available and present test results and the existing classical solutions. The current design rules for thin-cylinders in international steel specifications are also compared, and the newly derived design curve is proposed which was found in a good agreement with the available test results.

1. Introduction

1.1. Paradox in slenderness limits

There is presently a paradox in the difference between the static yield limits λ_{ey} and λ_y for tubes and cylinders under axial and bending loads, respectively. λ_{ey} is used in AS 4100 [1] to define a fully effective section under uniform axial compression, while λ_y is used to define a slender section that fails by elastic buckling under pure bending.

Table 1 shows that a number of design codes adopts $\lambda_y/\lambda_{ey}=1.0$ such as DIN 18800 [2], AIJ [3], BS 5950 [4], and Eurocode 3 [5]. However, ANSI 360-10 [6] adopts $\lambda_y/\lambda_{ey}=2.75$. AS 4100 [1] adopts $\lambda_y/\lambda_{ey}=1.46$. Elchalakani et al. [7] based on an extensive experimental testing derived $\lambda_y/\lambda_{ey}=1.71$. It is the author's opinion that the discrepancies in the yield limit ratio (λ_y/λ_{ey}) values between the design rules are resulting from different assumptions made in the developed theories and presentation of experimental results for elastic buckling of tubes and cylinders. It is also the author's opinion that the condition of $\lambda_y/\lambda_{ey}=1.0$ adopted in a number of steel specifications is unnecessary and very conservative as it will be further discussed in the paper.

A thick-walled cylinder under uniform axial compression fails by forming the well-known elephant foot, whereas it ovalises under pure bending in the inelastic range and then it fails by forming a smooth kink in the plastic range. On the other hand, a thin-walled cylinder fails by forming the diamond mode around the whole circumference (Fig. 2a), whereas it only forms such mode in the compression zone under pure bending. The main differences between the two types of loading are the presence of stress gradient under pure bending and the number of half-waves formed in the circumferential direction. The stress gradient shown in Fig. 2b has two effects on the behaviour within the elastic range. First, it relatively reduces the total force applied to the buckles, hence it limits the zone of instability and delays buckling. Second, the buckles under bending are comparatively retrained by adjacent lesser stressed fibres. This restraining action is significant compared to the symmetric diamond mode under uniform axial compression (Fig. 2b), where all the fibres are stressed to the same degree. Such comparatively large restrained fibres could explain why the experimentally derived ratio λ_y/λ_{ey} is often found more than 1.0 in the tests [7].

1.2. Past theoretical investigations

Table 2 lists the previous known closed-form solutions for elastic and inelastic buckling for thin-cylinders under pure bending.

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Nomenclature

AIJ	Architecture Institute of Japan
a	elliptical ripple major dimension
a_m	amplitude of displacements
a_n	amplitude of displacements
b	elliptical ripple minor dimension
CHS	circular hollow sections
D	mean diameter of the tube
E	modulus of elasticity
E_T	tangent modulus of steel tube
H	flexural rigidity of the tube wall
K	buckling coefficient
K_m	buckling coefficient for cylinders with medium slenderness
K_n	Buckling coefficient for cylinders with large slenderness
K_{0m}	buckling coefficient for cylinders with medium slenderness
K_{0n}	buckling coefficient for cylinders with large slenderness
L	total length of tube
L_m	more Fringe film length
L_{AB}	distance between right and left wheels
M_{cr}	critical elastic buckling moment
N_0	critical maximum axial compressive force per unit length
N_x	non-uniform axial compressive force per unit length
n	number of half-waves within ellipse in hoop direction in Chapter 7
R	mean radius of tube used in Chapter 2
R_{lw}	radial dimension at left loading wheel
R_{rw}	radial dimension at right loading wheel
t	thickness of cylinder
W_{int}	internal work dissipated in the mechanism
W_{ext}	external work done by the applied moment
x, y, z	rectangular coordinates
Z	elastic section modulus
Z_e	effective section modulus
α_l' and α_r'	angular change at loading wheel
ΔD	initial out of round, or average ovalisation
ΔU	change in strain energy due to bending
ΔW	change in work done by compression force
ε_{cr}	critical strain
γ_c	central arc angle defined in Chapter 7
γ_l' and γ_r'	left and right jack angles
κ_{cr}	critical curvature
χ	strength reduction factor
χ_0	coefficient
λ_s	section slenderness defined in AS 4100
λ_y	yield slenderness limit for pure bending
λ_{ey}	yield slenderness limit for axial compression
λ_s	$(D/t)(\sigma_y/250)$
ν	Poisson's ratio
σ_{cr}	critical buckling stress under axial compression
σ_a	critical buckling stress under uniform axial compression
σ_y	yield stress of steel tube
σ_b	critical buckling stress under pure bending
ΣW_i	sum of the elastic work
θ	relative angle of rotation

Flugge [8] used an equilibrium method and performed a bifurcation buckling analysis of thin-walled tubes. He assumed small deflections and membrane pre-buckling stresses to derive the buckling equations. His results showed that the critical buckling stress under pure bending is 33% larger than the corresponding one under uniform axial compression ($\sigma_b/\sigma_a = 1.33$). This corresponds to a yield limit ratio of $\lambda_y/\lambda_{ey} = 1.33$.

Seide and Weingarten [9] also used an equilibrium method and assumed small deflections to derive the critical buckling stress for an infinitely long tube subjected to pure bending. They used Batdorf's modified Donnell's differential equation for buckling and the applied Galerkin's method to derive the stability criteria to solve the differential equation under a stress gradient due to bending. Their results showed that the ratio of σ_{cr} under axial compression to the corresponding one under pure bending is essentially equal ($\sigma_b/\sigma_a = 1.0$). This corresponds to a yield limit ratio of $\lambda_y/\lambda_{ey} = 1.0$. Karyadi [10] carried out finite element analyses on thin-walled tubes where $D/t = 200$ and subjected to both uniform axial compression and bending. He studied the effect of length variation on the elastic buckling stress where $L/D = 0.1$ to 5. His numerical results showed that the stress ratio σ_b/σ_a non-linearly varies with the length of the tube. He showed that $\sigma_b = 1.07\sigma_a$ for very short tubes where $L/D = 0.1$, whereas $\sigma_b = 1.02\sigma_a$ for relatively longer tubes where $L/D = 2.5$. Murray and Bilston [11] performed a non-linear buckling analysis for non-compact tubes that fail by forming the ripples in Fig. 3a. They derived equations based on the well-known "Beam-on-Elastic Foundation" model and found that $\sigma_b = \sigma_a$ if the elastic modulus

Table 1
Yield slenderness limits for CHS in international codes [$\lambda = (D/t)(\sigma_y/250)$].

Country	Code/investigators	Yield limit ratio λ_y/λ_{ey}	Axial compression yield limit λ_{ey}	Pure bending	
				Plastic limit λ_p	Yield limit λ_y
Australia	AS 4100 [1]	1.46	82	50	120
	Elchalakani et al. [7]	1.71	82	60	140
New Zealand	NZS 3404 [26]	1.46	82	50	120
Canada	CAN/CSA S 16.1 [33]	1.0	92	72	92
Germany	DIN 18800, Part 1 [2]	1.0	84	65	84
Japan	AIJ [3]	1.0	94	N/A	94
Belgium	NBN 51-002 [27]	1.0	94	65	94
United Kingdom	BS 5950, Part 1 [4]	1.0	88	62	88
Europe	Eurocode 3, Part 1.1 [5]	1.0	84	65	84
USA	ANSI 360-10 [6]	2.75	91	57	250
	Sherman [15]	2.75	91	57	250