Thin-Walled Structures 80 (2014) 153–158

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Homogeneous generalized yield criterion based elastic modulus reduction method for limit analysis of thin-walled structures with angle steel

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ARTICLE INFO

Article history:
Received 25 January 2013
Accepted 28 February 2014
Available online 9 April 2014

Keywords:
Limit analysis
Homogeneous generalized yield criterion
Elastic modulus reduction method
Thin-walled structures
Angle section

ABSTRACT

A homogeneous generalized yield criterion (HGYC) expressed by piecewise polynomial is given for angle section. The element bearing ratio (EBR), reference EBR, and uniformity of EBR are defined in term of the HGYC. Then, a HGYC based elastic modulus reduction method for limit analysis of thin-walled structures with angle steel is presented based on the modulus adjustment strategy established on the EBR and the conservation criterion of energy. The method proposed can overcome the disadvantage of the conventional elastic modulus adjustment procedure where the nonhomogeneous generalized yield criterion is employed and the proportional loading condition is not satisfied.

1. Introduction

The beam, plate and shell element are often used in finite element method to discretize complex thin-walled structures for balancing the efficiency and accuracy, so that it is not necessary for the cross section of structural components to be discretized into fine mesh. When using these elements with large size to perform limit analysis, a key step is to find the exact generalized yield criterion (GYC) for cross section concerning its characteristic geometric configuration. Much effort has been devoted to develop GYCs for component's cross sections with different kinds of geometric profiles, especially on symmetrical sections. Duan and Chen [1] among others proposed a general four-parameter three-dimensional GYC for doubly symmetrical sections. Yang and Hung [2] developed a five-dimensional GYC by three component yield surfaces, which included the effect of torsion and bimoment for I-shape section. Gendy and Saleeb [3] utilized all seven sectional internal forces to derive two GYCs approximately for rectangular and wide I-shape sections. Burgoyne and Brennan [4] and Robinson [5] had re-evaluated several GYCs for thin shells.

Being an extensively used steel member in thin-walled structures, angle steel is commonly considered as a kind of monosymmetric or even non-symmetrical section, which makes it more difficult to establish its GYC. Chen and Atsuta [6] presented a yield surface curve with combined axial force and biaxial bending moment by numerical method. Based on the virtual work principle and some assumptions, Ho and Xie [7] gave lower and upper bound GYC solutions for those components under biaxial bending and axial load. Al-Bermani and Kitipornchai [8,9] developed two GYCs by 'trial and error' method through a numerical curve-fitting technique. By introducing some assumptions like material being concentrated at the centerline of the section, stress discontinuity line being placed always passing the two legs simultaneously, Cho and Chan [10] derived a GYC for angle section subjected to biaxial bending and axial force. After taking away the limitation of the position of stress discontinuity line, Vayas and Charalampakis [11,12] presented a more general GYC with biaxial bending and axial force. More recently, Aydin [13] evaluated the interaction relationship of internal forces through their experiments. These GYCs for angle section are all formulated with nonhomogeneous function.

As a kind of important numerical method for limit analysis by linear elastic iteration analysis, the elastic modulus adjustment procedure (EMAP) was developed quickly in the past 25 years. Many different EMAPs were proposed, such as GLOSS method [14], elastic compensation method [15], linear matching method [16], m_α-tangent method [17], elastic modulus reduction method (EMRM) [18]. Besides, elastic plastic incremental method (EPIM) verified by many model tests is also commonly used for ultimate bearing capacity analysis without regard to its relatively low computational efficiency [19,20]. The GYCs for beam, plate and

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http://dx.doi.org/10.1016/j.tws.2014.02.030
0263-8231

shell have also been introduced into the EMAP and EPIM [21–24] for improving calculation efficiency. Before moving forward, the proportional loading condition, say load effect varying linearly with corresponding load, should be satisfied according to limit analysis theory. However, the existing GYCs are mainly in nonhomogeneous form because of the complex interaction relationship among internal forces. If the influence of certain internal force on limit load is dominated, satisfying results can be achieved by the EMAP combined with those GYCs for limit analysis. However, when more than one internal force needs to be considered, some problems, such as variability of result with different initial loads and bad accuracies, will be encountered because these nonhomogeneous GYCs cannot meet the proportional loading condition. In most cases axial force and bending moment in angle steel of thin-walled structures are equally significant to limit load. Therefore, an efficient EMAP needs to be developed combining with homogeneous GYC for limit analysis of thin-walled structures.

In this paper, a homogeneous generalized yield criterion (HGYC) in terms of piecewise polynomial is developed for angle section by employing regression analysis and least square method. The order of the polynomial expression is determined by error analysis for balancing the calculation efficiency and accuracy. The EBR, reference EBR, and uniformity of EBR based on the HGYC are then defined for providing a dynamic criterion to identify the elements with higher EBR. Then, a HGYC based elastic modulus reduction method for limit analysis of thin-walled structures, named HGYC-EMRM, is presented based on the modulus adjustment strategy established on EBR and the conservation criterion of energy. The HGYC-EMRM overcomes the problem encountered in the existing EMAPs with nonhomogeneous GYCs where the proportional loading condition is not satisfied.

2. Homogeneous generalized yield criterion for angle section

2.1. GYC for angle section

Considering an equal angle section shown in Fig. 1, the section is idealized by assuming that the material is concentrated at the centerline of its two legs and the actual round corners are replaced by sharp corners. Axis y and z are the two inertia axis.

The full axial plastic force N_{px}, and the two full plastic moments M_{py} and M_{pz} are determined as follows [10]:

$$N_{px} = \sigma_s A, \quad M_{py} = \frac{\sqrt{2}}{2}\sigma_s b^2 t, \quad M_{pz} = \frac{\sqrt{2}}{4}\sigma_s b^2 t \tag{1}$$

where σ_s is material yield strength. b and t are the idealized length and thickness of the legs, respectively.

The GYC for angle section proposed by Charalampakis [12] is

$$f = \begin{cases} n_x^2 + m_z + \rho\left(\frac{|m_y|}{1+n_x}\right)^{2-|n_x|} \le 1, & 2n_x(1-|n_x|) < m_z \le 1-n_x^2 \\ n_x^2 - m_z + \rho\left(\frac{m_y}{1-n_x}\right)^{2} \le 1, & n_x^2 - 1 < m_z \le 2n_x(1-|n_x|) \end{cases} \quad (n_x < 0) \tag{2}$$

$$f = \begin{cases} n_x^2 + m_z + \rho\left(\frac{m_y}{1+n_x}\right)^{2} \le 1, & 2n_x(1-|n_x|) < m_z \le 1-n_x^2 \\ n_x^2 - m_z + \rho\left(\frac{|m_y|}{1-n_x}\right)^{2-|n_x|} \le 1 & n_x^2 - 1 < m_z \le 2n_x(1-|n_x|) \end{cases} \quad (n_x \ge 0) \tag{3}$$

where n_x, m_y and m_z are non-dimensionalized internal forces, obtained through $n_x = N_x/N_{px}$, $m_y = M_y/M_{py}$, $m_z = M_z/M_{pz}$. ρ is defined as

$$\rho = (1-|n_x|)(1+3|n_x|)/(1+|n_x|)^{2-|n_x|} \tag{4}$$

The domain of m_y is

$$n_x^2 - 1 \le m_y \le 1 - n_x^2 \tag{5}$$

2.2. HGYC for angle section

Though the nondimensionalized inner forces increase proportionally with the external load, there exists a non-proportional relationship between the external load and the generalized yield function (GYF) f defined in Eqs. (3) and (4), which does not match the proportional loading condition claimed by limit analysis theory. In order to introduce this GYF into EMAPs and perform limit analysis of thin-walled structures with angle steel, a corresponding homogeneous GYF (HGYF) needs to be provided to satisfy the proportional loading condition. Here, a HGYF fitted by a piecewise function composed of a set of homogeneous polynomials is investigated:

$$\bar{f}_R(n_x, m_y, m_z) = \sum_{i=1}^{H} a_i n_x^j m_y^g m_z^{R-j-g}$$
$$(i = 1, \ldots, H;\ j = 0, \ldots, R;\ g = 0, \ldots, R-j) \tag{6}$$

where H, R and a_i are the number of polynomial terms, power and coefficient of every term, respectively.

The least square method is employed and the fitting criterion is

$$||\delta||_2^2 = \min \sum_{i=1}^{N} [f'(n_x^i, m_y^i, m_z^i) - f(n_x^i, m_y^i, m_z^i)]^2 \tag{7}$$

where (n_x^i, m_y^i, m_z^i) are the collocation points selected.

A HGYC can be established based on Eq. (6) and the collocation points. The biquadratic HGYF, say R=4, can be

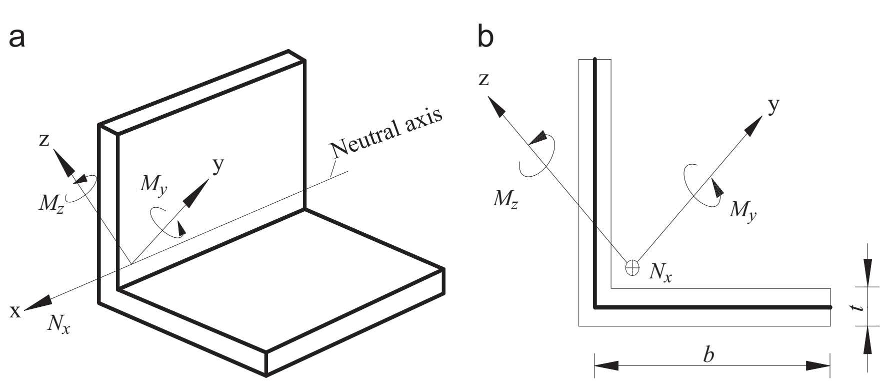

Fig. 1. Angle section: (a) internal forces and (b) idealization of equal angle section.