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Exact stress functions implementation in stability analysis of plates with different boundary conditions under uniaxial and biaxial compression

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ABSTRACT

Analytical approach used for critical load determination is based on Ritz energy technique in which two factors are crucial for the accuracy of results. First factor is deflection function. Herein, double Fourier series are used to represent buckled shape of the plates under arbitrary external loads. The second and the most important factor is adoption of adequate realistic stress distribution within plate prior to buckling. Based on the Baker&Pavlović&Tahan and Pavlović&Liu investigations, exact stress distributions were introduced herein. In that way, with the adequate deflection functions and precise stress distribution, Ritz energy method can produce highly accurate results.

Through several examples for the plates with different boundary conditions, for which available literature offers very few analytical solutions, accuracy of the presented analytical approach is proved. Results obtained by analytical approach are reaffirmed by numerical finite-element runs.

1. Introduction

Previous studies on stability of rectangular plates under influence of variable loads were mostly based on assumptions of simplified stress distribution, which led to questions as the accuracy of results thus obtained. This paper shows how the stability of plates with mixed boundary conditions under arbitrary in-plane load distributions, analyzed up to now almost exclusively by numerical methods, can be tackled analytically in an accurate, practically exact manner. The proposed method uses an approach based on Ritz energy technique. Introduction of the exact two-dimensional elasticity solutions for in-plane stress distributions is crucial contribution and main condition for the wide applicability of the presented analytical method. By adopting exact stresses within a plate under any type of external loads and by using the double Fourier series to represent any possible buckled profile for plates with different boundary conditions, the critical loads can be obtained in very accurate way. The results produced, for the first time, by the analytical approach proposed in this article, are compared to those from numerical finite element method.

It is interesting that the analytical procedure for determining the exact stress distribution within a rectangular plate, loaded by in-plane arbitrary external load is based on the solution dated from XIX century. In fact, in 1890 Mathieu [1] was the first to determine the stress function for the particular case of a rectangular plate loaded along the edges by an arbitrary compressive stress through superposition of two (DEA and DEB), so called, basic types of load (Fig. 1).

In the series of papers by Baker and Pavlović [2–5], one of these types of load (DEA), was applied during the stability analysis of simply supported plates, loaded by the locally distributed compressive stress. Several years later, in 1993, more than one century after the Mathieu's original paper had appeared, the same authors returned to the basic problem [6]. Their aim was to obtain the solution of the exact stress distribution for the general case of the rectangular plate loaded by the arbitrary external in-plain load. The fact is that any in-plane load (normal and/or shear) which acts along the edges of the plate, can be described by the chosen functions (even and/or odd in relation to the coordinate axes), so the total solution is obtained by the adequate combination of 8/16 basic load cases (eight for each of two orthogonal directions) (Fig. 2).

To enable a detailed description of the analytical procedure for critical buckling load determination for the plates with different boundary conditions, the basic load case DEA is used in this paper. So far, according to the literature, this kind of analysis has been done almost exclusively for simply supported plates [2–5,7–9] with very few exceptions [10–12].

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Notation

a	plate length (measured along the x-axis)
A_o, A_n, A_m	Fourier-series coefficients for arbitrary external loadings $f(y)$, $f(x)$
b	plate width (measured along the y-axis)
B_n, B_0	unknown group of coefficients defined by Eqs. (13) and (14)
$D=Et^3/12(1-\nu^2)$	flexural rigidity of the plate
$e(x)=\sinh(x)$	abbreviation
$E(x)=\cosh(x)$	abbreviation
E	Young's modulus
$f(x)$, $f(y)$	functions defining the applied load on the edges $y=\pm b/2$ and $x=\pm a/2$
F, $(F=F_1+F_2)$	function introduced to satisfy Eq. (6)
G_m	unknown group of coefficients defined by Eq. (12b)
H_n	unknown group of coefficients defined by Eq. (12a)
l_1, l_2	load length on the edges $x=\pm a/2$ and $y=\pm b/2$ respectively
K	buckling coefficient for the case of distributed load
K_T	buckling coefficient for the case of concentrated force
N_1, N_2	analytical solutions for direct stresses along x and y axis respectively
N_{min}	minimal number of terms in one direction for deflection function
p_x, p_y	load intensity in x and y direction respectively
P	intensity of the concentrated force
t	thickness of the web
T_3	analytical solution for shear stress in the plane x-y
u, v	displacements along the x and y directions respectively
U	strain energy due to bending
V	potential energy associated with the work done by external loads
w	out-of-plane deflection function
W_{mn}	coefficients for deflection function
$\alpha=(\lambda+2\mu)/\mu$	constant defined in terms of Lame's parameters
β_m, β_0	unknown group of coefficients defined by Eqs. (13) and (14)
$\gamma_x=l_1/b$	patch-load ratio on the edges $x=\pm a/2$
$\gamma_y=l_2/a$	patch-load ratio on the edges $y=\pm b/2$
Δ	Laplase's operator
$\varepsilon=\mu/(\lambda+\mu)$	constant defined in terms of Lame's parameters
$\lambda=\nu E/(1+\nu)(1-2\nu)$	Lame's parameter
Λi	functions in non-dimensional form defined by Eq. (15)
$\mu=E/2(1+\nu)$	Lame's parameter
$\boldsymbol{\nu}$, $(\boldsymbol{\nu}=\boldsymbol{\nu}_1+\boldsymbol{\nu}_2)$	volumetric dilatation
ν	Poisson's ratio
Π	total potential energy of the system
σ_x, σ_y	direct stresses
τ_{xy}	shear stress in the plane x–y
$\tau(x)=E(x)-x/e(x)$	abbreviation
$\phi=a/b$	aspect ratio of the plate
$\Psi(x)=e(x)/\tau(x)$	abbreviation

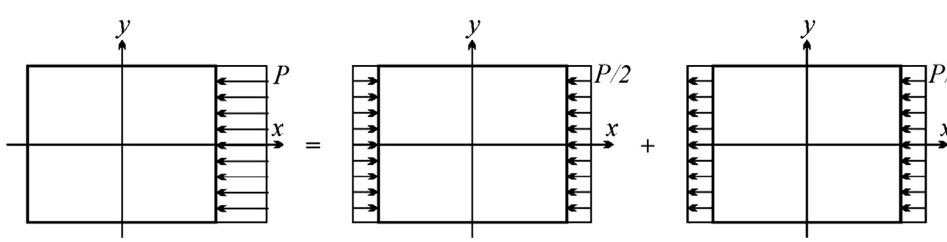

Fig. 1. Decomposition of loads (cases DEA and DEB).

2. Basic outline

Before proceeding with solution, it is necessary to summarize the main governing expressions of two-dimensional elasticity, as, in common with much of XIX century work, Mathieu's notation and approach depart from current conventions.

In his paper [1], Mathieu (1890) expressed the known equilibrium equations, without the presence of body forces, in terms of displacements

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0 \quad \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0$$
$$\text{Mathieu} \quad \Rightarrow \quad \Delta \mathrm{u}=-\frac{1}{\varepsilon}\frac{d\nu}{dx} \quad \text{and} \quad \Delta \mathrm{v}=-\frac{1}{\varepsilon}\frac{d\nu}{dy} \tag{1}$$

where $\boldsymbol{\nu}$ is volumetric dilatation given by

$$\boldsymbol{\nu}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}. \tag{2}$$

Using relatively simple mathematical operations, the equations from the system (Eq. (1)) can be written as follows:

$$\Delta\boldsymbol{\nu}=0. \tag{3}$$

Mathieu's approach to the 2D elasticity problem starts with the careful selection of two ordinary Fourier series for $\boldsymbol{\nu}$ (Eq. (3)) with infinite unknown coefficients, taking into account the symmetry or anti-symmetry of the stresses with respect to the x and y directions

$$\boldsymbol{\nu}=\boldsymbol{\nu}_1+\boldsymbol{\nu}_2. \tag{4}$$

The next step presents the introduction of the function F, $(F=F_1+F_2)$, from the conditions that the following equation is fulfilled

$$\Delta F=-\frac{1}{\varepsilon}\boldsymbol{\nu} \quad \Rightarrow \quad \Delta F_1=-\frac{1}{\varepsilon}\boldsymbol{\nu}_1 \quad \text{and} \quad \Delta F_2=-\frac{1}{\varepsilon}\boldsymbol{\nu}_2. \tag{5}$$

Finally, when displacements u and v are determined as follows

$$\mathrm{u}=\frac{dF}{dx}+\alpha\int\boldsymbol{\nu}_1 dx \quad \textit{and} \quad \mathrm{v}=\frac{dF}{dy}+\alpha\int\boldsymbol{\nu}_2 dy, \tag{6}$$

direct stresses N_1 and N_2 along axes x and y, as well as the in-plane shear stress T_3, are defined by Eqs.(7a) and (7b).

$$N_1=\lambda\boldsymbol{\nu}+2\mu\alpha\boldsymbol{\nu}_1+2\mu\frac{d^2F}{dx^2}, \quad N_2=\lambda\boldsymbol{\nu}+2\mu\alpha\boldsymbol{\nu}_2+2\mu\frac{d^2F}{dy^2} \tag{7a}$$