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# A model for warping transmission through joints of steel frames

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ABSTRACT

A simple approach is developed in this paper which considers the effect of partial warping continuity through the joints of thin-walled steel frames when using beam finite element analysis. Using a condensed stiffness matrix for the joint generated by the substructuring technique, warping springs are introduced to represent the condition of partial warping restraint at intersections between members. The performance of the proposed model is demonstrated through a number of numerical examples. Excellent agreement is achieved between the results of beam finite element analysis using the suggested joint model and accurate shell finite element analysis.



## 1. Introduction

Thin-walled steel frames with slender open-section members may experience significant torsion and therefore large warping displacements under applied loads. The warping deformation is generally defined as the longitudinal displacement caused by torsion. For a doubly symmetric I-section, the warping displacement consists of linear longitudinal displacements in the flanges in opposite directions.

The boundary conditions for warping at the ends of an isolated member can be divided into three main groups: completely free, fully restrained or partially restrained. The most important contributions in this context have been made by Timoshenko [1,2], Wagner [3] and Vlasov [4] who studied the warping (or non-uniform) torsion of I-beams and derived a general theory for thin-walled members. In the numerical implementation of the theory, many researches [5–7] introduced the first derivative of the twist rotation as the seventh degree of freedom to represent warping deformation. Toward this objective, conventional $12 \times 12$ stiffness matrices were replaced by the new ones with warping considered as an additional degree of freedom. In these studies, the end warping condition was assumed to be either completely free [8–10] or fully restrained [11–13] at both ends of the member.

The flexural–torsional behaviour of plane frames has been studied by numerous investigators, but in most cases either warping at joints was neglected by assuming six degrees of freedom for beam elements [14], or considered to be fully prevented [15]. In subsequent studies, the focus shifted from isolated members to frames by realising that at joints, warping displacements in one member may redistribute and produce warping and twisting in other connected members. This implies that when one member warps, the flanges of adjoining members must rotate, causing a distortion of the cross-section. Thus, the resistance of adjoining members to distortion provides a level of restraint on the warping torsion of the loaded member. Several experiments were conducted to determine the warping restraint at member ends [16,17]. All experiments reported the difficulties of restraining warping and demonstrated that even very stiff end connections do not provide full torsional warping restraint. Consequently, the concepts of continuous warping and partially restrained warping were introduced.

Austin et al. [18] studied the subject of elastic end warping restraint but no information was given to evaluate the degree of restraint. Trahair [19] introduced the ratio between the elastic flange and the fixed-ended flange moments as the degree of warping restraint. Tong et al. [20] suggested a model for warping transmission based on modifying the traditional thin-walled beam element matrix at the joints. Ettouney and Kirby [21] proposed a warping restraint factor, which is the ratio between the bimoments of the partially and fully restrained cases similar to the warping "spring" concept introduced by Yang and McGuire [22]. For both studies, static condensation was used to eliminate undesired degrees of freedom. Although the basic idea of the two methods was same, the Yang and McGuire's procedure seems to be more representative of partial warping restraint between two members as it operates with the warping deformation which is easier to measure than the bimoment. The model featured a hypothetical warping rigidity applied as an internal spring at the joint.

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The warping and distortion at angle joints composed of two steel I-section members of the same cross-section were investigated by Vacharajittiphan and Trahair [23]. Four types of joints (unstiffened, diagonal-stiffened, box-stiffened and diagonal/box-stiffened joints) were considered in that study for which the numerical values of end warping restraint were calculated [24]. The study concluded that warping and distortion are interdependent and depend on the joint configuration details. Subsequently, these particular types of joints composing two or more channels or I-sections were studied by Sharman [25], Morrel et al. [26] and Massarira [27] to determine the effect of warping transmission through joints (Fig. 1). An important contribution to the research on the transfer of warping through joints was presented by Basaglia et al. [28–31] using a numerical model which considered transmission of warping torsion and local displacement compatibility at frame joints of various configurations. The results of the model were compared with shell element FE analysis using ANSYS and excellent agreement was achieved.

A brief review of the literature on the transmission of warping through joints of thin-walled steel frames shows that all suggested models need substantial numerical or computational effort. Due to the complexity of current models, the partial transmission of warping through joints is ignored in most design cases. Even if a designer wanted to consider transmission of warping through joints, available commercial finite element software packages are limited to either completely prevent warping or allow warping to occur freely at joints when using beam finite element analysis (B-FEA). At this point in time, there appears to be no FE software available that allows the seventh degree of freedom (warping) to be partially transmitted. The only option to model warping accurately is using shell elements (S-FEA), which is not a desired method for complex structures due to its high computational cost. A few models can be found in the literature for the partial transmission of warping at joints when using beam finite elements [30,32].

Basaglia et al. [30] developed a simple kinematic model to simulate the warping transmission or restraint at the joints of thin-walled frames in the context of beam finite element analysis. The model relies on the facility of most structural analysis software (e.g. ABAQUS and ANSYS) to impose “linear constraint equations” which establish constraint conditions between the torsion warping degrees of freedom of the member end nodes. Despite its simplicity, the model is only applicable to four specific types of joints (see Fig. 1) and cannot model all possible cases with partially restrained warping. For example, the fully prevented warping assumption implied for diagonal/box stiffened joints may be rather conservative if the stiffeners have relatively small thickness. Also, the method does not extend to 3D joints with members adjoining in three orthogonal planes, as in common 3D frames.

The present study is concerned with developing a new method for modelling joints including warping effects. The method benefits from the accuracy of 3D shell finite element modelling and from the computational efficiency of using 1D beam elements. The new joint model is developed using a combination of the substructuring technique and linear springs. In the model, the joint accepts warping deformations from adjoining loaded members and redistributes the deformation to all connected beams and columns. In fact, the suggested joint model acts as a flexible interface between members and provides partial warping restraint by means of springs. The model is general and can be applied to any kind of joint in 2D and 3D thin-walled structural frames.

## 2. Substructuring and static condensation

Substructuring is a technique commonly used to overcome the difficulty of working with large dimensional problems [33]. In principle, a structure can be subdivided into smaller parts and each part analysed separately. The basic idea of substructuring analysis is that only certain degrees of freedom are retained while others are eliminated by static condensation. This methodology is available in many finite element software packages and offers many advantages: (i) a substantial reduction in analysis time is achieved by modelling only the joints using 3D shell finite elements rather than the entire frame, (ii) the substructure stiffness matrix needs only to be computed once for each type of joint with similar geometry, and (iii) by writing a script to generate the substructure, the stiffness matrix can be calculated automatically and there is no need to create the joint manually when changing the geometry of the joint.

According to the conventional finite element method, the global stiffness matrix $\mathbf{K}$ is obtained by assembling the stiffness matrices of all elements. The global stiffness equation can be expressed as

$$\mathbf{K}\mathbf{u} = \mathbf{F} \tag{1}$$

where $\mathbf{K}$ is an $n \times n$ matrix, $\mathbf{u}$ and $\mathbf{F}$ are $n \times 1$ node-displacement and load vectors respectively and $n$ is the total number of degrees of freedom. In the substructure analysis, Eq. (1) is modified to

$$\mathbf{K}^{*}\mathbf{u_r} = \mathbf{F}^{*} \tag{2}$$

where $\mathbf{K}^{*}$ and $\mathbf{F}^{*}$ are the stiffness matrix and force vector respectively of smaller dimension than $n$ obtained after static condensation. To obtain Eq. (2), in first step, the displacement

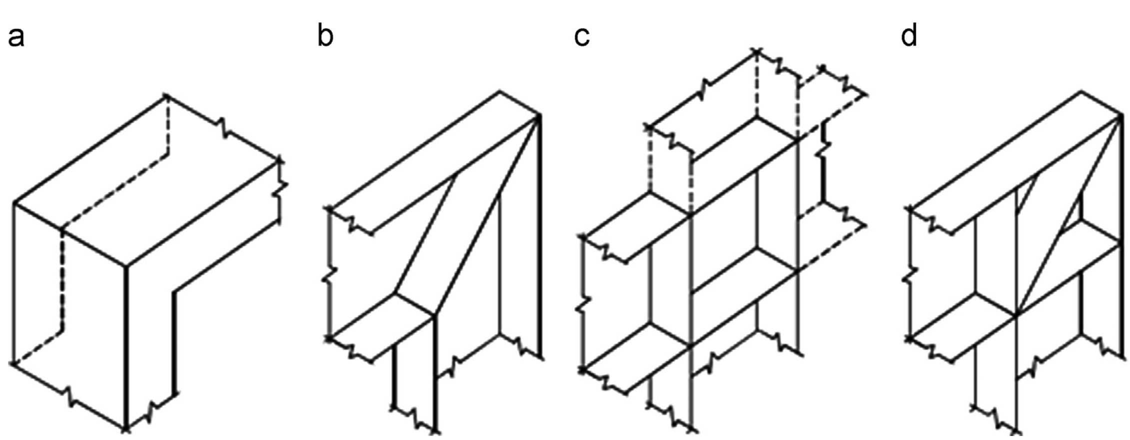


**Fig. 1.** Configurations of joints between channel members [28]: (a) unstiffened with flange continuity, (b) diagonal stiffened, (c) box stiffened, (d) diagonal/box stiffened joints.