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Effect of interfacial imperfection on bending behavior of composites and sandwich laminates by an efficient C^0 FE model

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ABSTRACT

The bending behavior of composites and sandwich plates having imperfections at the layer interfaces is investigated by a refined higher order shear deformation plate theory (RHSDT) and a Least Square Error (LSE) method. In this theory, the in-plane displacement field is obtained by superposing a globally varying cubic displacement field on a zig-zag linearly varying displacement field. This plate theory represents parabolic through thickness variation of transverse shear stresses which satisfy the inter-laminar continuity condition at the layer interfaces and zero transverse shear stress condition at the top and bottom of the plate. In this plate model, the interfacial imperfection is represented by a liner spring-layer model. Finite element method is adopted and an efficient C^0 continuous 2D finite element (FE) model is developed based on the above mentioned plate theory for the static analysis of composites and sandwich laminates having imperfections at the layer interfaces. In this model, the first derivatives of transverse displacement have been treated as independent variables to circumvent the problem of C^1 continuity associated with the above plate theory (RHSDT). The LSE method is applied to the 3D equilibrium equations of the plate problem at the post-processing stage, after in-plane stresses are calculated by using the above FE model based on RHSDT. The proposed model is implemented to analyze the laminated composites and sandwich plates having interfacial imperfection. Many new results are also presented which should be useful for the future research.

1. Introduction

Composites and sandwich structural components are widely used in mechanical, aerospace, civil, and other engineering fields due to their advantage of high stiffness and strength to weight ratio. However, these structures are weak in shear due to their low shear modulus compared to extensional rigidity as well as variation of material properties between the layers. Thus the effect of shear deformation is quite significant which may lead to failure.

The structural behavior of composite laminates cannot be predicted by classical plate theory [1] which under-predict the displacements and over-predict the natural frequencies and buckling loads. This kind of approach is not suitable for the analysis of laminated plates as the shear deformation is neglected in the formulation. In addition to that, the problem becomes much more complex if some inter-laminar imperfection is there in the form of weak bonding and the accurate evaluation of transverse shear stresses becomes difficult using 2D models available in the literature. In this context, a number of plate theories have been developed where the effect of shear deformation is considered in a refined manner.

The commonly used displacement-based 2D plate theories may be categorized into two groups and they are (1) equivalent single-layer plate theory (ESLT) and (2) layer-wise plate theory (LWT). In equivalent single-layer theory [2–7] the deformation of the plate is expressed in terms of unknown parameters of a single plane, which is usually taken as the middle plane of the plate. These are similar to Reissner–Mindlin's plate theory (i.e., the first-order shear deformation theory, FSDT) which requires shear correction factor but there are some improvements, which allow the warping of plate sections to have a higher-order variation of transverse shear stresses/strains along the thickness. In this context, the first-order shear deformation theory [7] may be considered as the simplest option where an arbitrary shear correction factor is used since the transverse shear strain is assumed to have uniform variation over the entire plate thickness. The performance of the first-order shear deformation theory is dependent on the shear correction factor [6]. For a better representation of the transverse shear deformations, higher order plate theories (HSDT) are proposed by Lo et al. [2], Manjunatha and Kant [3], Reddy [4], and a few others, where the use of shear correction factor could be eliminated. It gives continuous variation of transverse shear strain across the entire thickness, which leads to discontinuity in the

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variation of the transverse shear stresses at the layer interfaces. But the actual behavior of laminated plate is the opposite i.e., the transverse shear stress is continuous at the interfaces whereas the strains may be discontinuous. Moreover, the degree of discontinuity in the transverse shear strain is severe especially for sandwich plates due to a wide variation in their material properties.

Considering this aspect in view and to model other important features of thick laminate in a better manner, a number of layer wise plate theories [8–11] have been developed where the deformation of the plate is expressed in terms of unknowns of a number of planes, which are taken at the layer interfaces and also at some intermediate levels in some cases. It gives a zigzag through the thickness variation of in-plane displacement, which represents the desired shear strain discontinuity at the layer interfaces. The performance of these plate theories is good but the mathematical involvement in these plate theories is quite heavy and the solution becomes quite expensive in a multilayered plate, as the number of unknowns is dependent on the number of layers. A major development in this direction is due to Di Sciuva [12], Murakami [13], Liu and Li [14], and few others. They proposed zigzag plate theory where layer-wise theory is initially used to represent the in-plane displacements having piecewise linear variation across the thickness. The unknowns at the different interfaces are subsequently expressed in terms of those at the reference plane through satisfaction of transverse shear stress continuity at the layer interfaces and the number of unknowns is dramatically reduced. A further improvement in this direction is due to Bhaskar and Varadan [15], Cho and Parmerter [16,17], Di Sciuva [18], and some other investigators who considered the variation of in-plane displacements to be a superposition of a piecewise linearly varying field on an overall higher order variation. Carrera [19], and Demasi [20] considered higher order terms in the displacement field, using Mukarmi's zig-zag function [13] and assumptions for transverse stresses brings about a large number of solution variables. However applying static condensation technique allows to eliminate the unknowns related to the transverse stresses and thus, to derive efficient plate theories [21,22]. Kapuria et al. [23,24] has presented zigzag theory for hybrid beams and plates in which the number of variables is reduced to FSDT by satisfying interface and boundary conditions, it yield approximately accurate results for cross ply only. Zhen and Wanji [25] have proposed C^0 type higher-order theory for bending analysis of laminated composite and sandwich plates. Subsequently, Zhen et al. [26] also proposed C^0 type finite element based on higher-order theory for accurately predicting natural frequencies of sandwich plate with soft core. These theories are usually referred as refined higher order shear deformation theory (RHSDT). However, there are very few C^0 elements reported in the literature which can model the RHSDT.

In all these refined higher order shear deformation theories, a perfect interface between the layers is assumed, which is characterized by continuous displacements and tractions across the interfaces. But in case of imperfect interfaces, there should be jumps in the displacement components at the interfaces whereas traction would remain continuous from an equilibrium point of view [27,28]. Di Sciuva and Gherlone [29,30] have developed an analytical as well as a FE model based on third-order Hermitian zig-zag plate theory to study the damaged bonded interfaces. The simplest way to model this phenomenon is to use a linear spring layer model where the displacement jumps in a particular interface are proportional to the tractions at that interface. Such an attempt has been made by Cheng et al. [31], Di Sciuva [32], Chakrabarti and Sheikh [33,39] and a few others where the linear spring layer model has been implemented in a plate model based on RHSDT.

Considering all these aspects in view, an attempt has been made to study the behavior of composites and sandwich laminates with inter-laminar imperfection of arbitrary variation at the different levels by modifying the FE model proposed by Singh et al. [34] based on the RHSDT in combination with the linear spring model of Cheng et al. [31]. An efficient C^0 finite element model based on RHSDT has been presented in this paper for the analysis of composites and sandwich laminates with inter-laminar imperfection. In this model, the in-plane displacement fields are assumed as a combination of a linear zigzag function with different slopes at each layer and a global cubically varying function over the entire thickness. The proposed model satisfies the transverse shear stress continuity conditions at the layer interfaces and the zero transverse shear stress condition at the top and bottom of the plate. The isoparametric quadratic plate element has nine nodes with seven nodal unknowns (i.e., in-plane displacements and transverse displacement at the reference mid surface, along with rotational degrees of freedom at the reference mid surface) at each node. In this model the first derivatives of transverse displacement have been treated as independent variables to circumvent the problem of C^1 continuity associated with the above plate theory (RHSDT).

In the present paper the global response (i.e. displacement) of laminated imperfect plates is calculated first, using the above mentioned C^0 FE model based on RHSDT and then an accurate prediction of transverse stresses is done by an efficient LSE method (Ref. Khandelwal et al. [35]) using the 3D equilibrium equations. It is interesting to note that in the proposed model, the displacement field used for the calculation of unknown displacements and in-plane stresses is consistent with the stress field chosen along with the condition of stress continuity at the layer interfaces for the calculation of transverse shear stresses based on the LSE method. It should be noted that the evaluation of transverse shear stresses is not possible in the case of imperfect laminates by following the conventional procedure to calculate them by using the constitutive relationship.

The proposed model is validated by solving different problems of composites and sandwich plates with perfect as well as imperfect interfaces, as there are very few results available of laminated sandwich plates with inter-laminar imperfections. Finally the proposed model is applied to the actual problem where numerical results are generated by solving a number of problems to study the behavior of the present structure under different situations.

2. Mathematical formulation

2.1. FE model for displacements and in-plane stresses evaluation

The in-plane displacement fields (Fig. 1) are typical to those of RHSDT and are as below:

$$u_\alpha = u_\alpha^0 + \xi_\alpha z^2 + \varphi_\alpha z^3 + (zT_\alpha^0) + \sum_{k=0}^{n_u-1}[S_\alpha^k(z-z_k) + \Delta u_\alpha^k]H(z-z_k^u)$$
$$+ \sum_{k=1}^{n_l-1}[T_\alpha^k(z-\rho_k) - \Delta u_\alpha^k]H(-z+\rho_k). \quad (1)$$

where the subscript α represents the co-ordinate directions [α=1, 2], and u_α^0 denotes the in-plane displacements (i.e., u_1^0 along x-axis and u_2^0 along y-axis) of any point on the mid surface, n_u and n_l are number of upper and lower layers respectively, S_α^k and T_α^k are the slopes of kth layer corresponding to upper and lower layers respectively, ξ_α and φ_α are the higher order unknowns and $H(z-z_k)$ and $H(-z+\rho_k)$ are unit step functions.