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Global static and stability analysis of thin-walled structures with open cross-section using FE shell-beam models

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ABSTRACT

The finite shell-beam models for static and global stability analysis of thin-walled structures with open cross-section are proposed. The discretization using thin-walled beam elements is connected with the space discretization of some parts of the frame. The *space joint element*, formulated using only translational degrees of freedom on cross-sections connecting the joint with the thin-walled beams and the so-called *transition elements*, defined between the beam and the shell nodes are used for consistent coupling beams and shell parts.

1. Introduction

The history of contemporary theory of thin-walled structures is joined with the name of Vlasov, who in 1959 formulated the main hypothesis of the theory [1], taking into account earlier achievements of such scientists as Timoshenko, Bach and Wagner, from the beginning of the twentieth century. However, in the analysis of thin-walled structures, for example plane or space frames, the research problems establishing real boundary conditions and conditions of frame joints connections were still not solved in the manner which could be satisfactory for engineering practice [2,3]. The effect of joint warping restrains on the stress distributions was tried to be passed over accepting additional assumptions or limiting the types of thin-walled frames considered. As an example, the concept of the so-called warping indicator by Yang and McGuire [4] can be mentioned. Gorbunov and Strieblicka [5] were probably the first to formulate some conditions which have to be satisfied by the special construction of frame joints in order to ensure warping continuity where the Vlasov theory could have been directly applied.

The Finite Element Method (FEM) created the new quality in the development of methods of thin-walled structures analysis, thus stimulating evolution of the thin-walled theory and ensuring the numerical analysis of large structure systems with great exactness. In general, two finite element procedures of analysis are possible. In the first case, the thin-walled structure is treated as the full space shell structure, which is natural approximation of the real structure geometry, but it can be laborious and costly. In the other case, however the beam nature of the structure is conserved with very advanced displacement theory of the thin-walled beam. This approach was derived in the finite element (FE) geometrically nonlinear analysis of thin-walled frames from the 70s of the twentieth century when Borsum and Gallagher elaborated the geometrical stiffness matrix for the thin-walled beam element with the seven degrees of freedom in the node [6]. In this line Reill [7], Waldron [8,9] papers and great number of other papers, for example Kim et al. [10–12], can also be mentioned. This direction of the FE analysis was summarized in Waszczyszyn, Cichoń and Radwańska monograph [13].

Later on, the more precise tangent stiffness matrices of thin-walled beam elements with open cross-section were prepared into the large displacement analysis of elastic and elastic–plastic structures using the total Lagrangian approach by Pi and Trahair [14] and using the updated Lagrangian approach by Conci and Gattas [15] and Chen and Blandford [16]. Izzuddin and Smith derived an element for large displacement analysis of elastic–plastic structures using the Eulerian approach in a local convected system [17,18]. The other approach to the global buckling analysis of plane and space frames using General Beam Theory (GBT) was formulated by Basaglia et al. [19]. Lastly, the GBT method was used for the analysis of sensitivity of imperfections for the thin-walled right angle frame [20]. In the authors' opinion, the different complicated beam elements were successfully derived but the

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problem of their effective applications to the static and stability analysis of the real structures with different joint constructions remained still open.

Inspiring suggestions, limited, however, only to the linear analysis of simple plane frames with I-beam cross-section, can be found in Szmidt [21] where the FE model is divided into two parts, namely 3D part (shell type) and 1D part (beam type) and the procedure of the joint stiffness matrix computation is presented. Later on, such approach was also used in Cichoń and Pluciński [22], Szymczak et al. [23], Wagner and Gruttman [24] and Chavan and Wriggers [25] and is derived in the paper.

In the paper, the global stability analysis is limited to the elastic structures with one parameter load only. The fundamental equilibrium paths in load-configuration space are calculated. Special case of the initial buckling analysis is also discussed. The detailed discussion of the special methods of calculations of critical points (bifurcation or limit points) and of post-critical equilibrium paths is beyond the scope of the paper and can be found elsewhere [13].

The outline of the paper is as follows. The basic ideas of the paper are presented in Section 2. In Section 3 the variational equilibrium equation for the elastic body in the total Lagrangian description is formulated. The incremental finite element equilibrium equations are given in Section 4. In Sections 6 and 7, some matrices and vectors necessary for the explicit calculations of the flat shell element and the thin-walled beam element are derived. In addition, in Section 7 the procedure of calculation of the tangent stiffness matrix and the internal force vector of the thin-walled beam element is described. Section 8 contains detailed description of the space joint element and in Section 9 the transition element is shortly described. Examples of static and stability analysis of plane and spatial thin-walled structures are given in Section 10, followed by some conclusions in Section 11. In three appendices more cumbersome formulae are given.

2. Problem formulation

Discretization with only thin-walled beam elements (*FE beam model*, *1D model*) is possible for joints where complete warping transmission is ensured. Plane frame joints, made of the beams at right angle, with the same cross sections provide complete warping transmission [5,33], Fig. 1. Spatial frames and frames consisting of beams with different cross-sections should be discretized with shell elements (*3D model*). Other approach is analysis using mentioned earlier GBT-based beam finite elements especially applied to the frames made of U-section and I-section beams. In authors' opinion, a good alternative can be *mixed FE shell-beam model* (*1D/3D model*).

The basic problem in the mixed FEM shell-beam model is how to integrate parts of the structure composed of the beam and shell elements in the one compact finite element model.

The simple method is exploiting condition of the translational displacements continuity on the walls, common for the shell and thin-walled beams:

$$\mathbf{u}^{(s)}(\mathbf{x}) = \mathbf{u}^{(b)}(\mathbf{x}), \tag{1}$$

where $\mathbf{u}^{(s)}(\mathbf{x})$ is displacement vector of the shell finite element node and $\mathbf{u}^{(b)}(\mathbf{x})$ is displacement vector of the thin-walled beam node.

In the paper, two methods of using Eq. (1) were used as it is illustrated in Fig. 2.

In the first method (Fig. 2(b)), Eq. (1) was used to elaborate the *space joint element*, which is assumed to be the welded structure discretized with the flat shell elements and with the degrees of freedom reduced to the thin-walled beam degrees of freedom. The reduction process is realized in two steps, namely in the static condensation step [26] and the transformation step, Fig. 3. As a result of the static condensation, degrees of freedom of the space joint element are reduced to the translational degrees of freedom only in the cross-sections, common for the space joint element and the thin-walled beam element. Finally, the translational degrees of freedom are transformed to the seven degrees of freedom of the proper thin-walled beam element nodes.

In the second approach (Fig. 2(c)), the so-called *transition elements* [24,25] have been used. The transition element joins the node of the thin-walled beam element with the node of the shell element with the continuity condition of translational displacements on the walls, common for the shell and thin-walled beams.

3. Variational equilibrium equations

Let us consider a deformable, three-dimensional body which corresponds to continuous structure, Fig. 4. Kinematic relations are defined in the total Lagrangian formulation [27], where all quantities are referred to the original, undeformed configuration of the structure C^0 with the initial volume V^0 and the boundary surface S^0. Kinematic and static boundary conditions are described on parts as S_u and S_t, respectively, where $S = S_u \cup S_t$ i $S_u \cap S_t = \varnothing$. Current configuration is marked by C^t and incremental configuration by $C^{t+\Delta t}$. The structure is under the action of body forces $\mathbf{g}$ and tractions $\mathbf{t}$. Finally, the original coordinate $\mathbf{x}$ is used to label a material particle P^0 in configuration C^0 and the coordinate $\mathbf{X}$ describes the position currently occupied by a material particle P^t in configuration C^t. Vector $\mathbf{u} = \mathbf{X} - \mathbf{x}$ is the displacement vector and $\Delta\mathbf{u} = \mathbf{u}^{t+\Delta t} - \mathbf{u}^t$ is the vector of displacement increments. In order to establish the finite element formulation, the virtual work equation in configuration $C^{t+\Delta t}$ is considered:

$$\delta W_i = \delta W_e, \tag{2}$$

where δW_i is the variation of the internal energy and δW_e is the variation of the work done by the external loads.

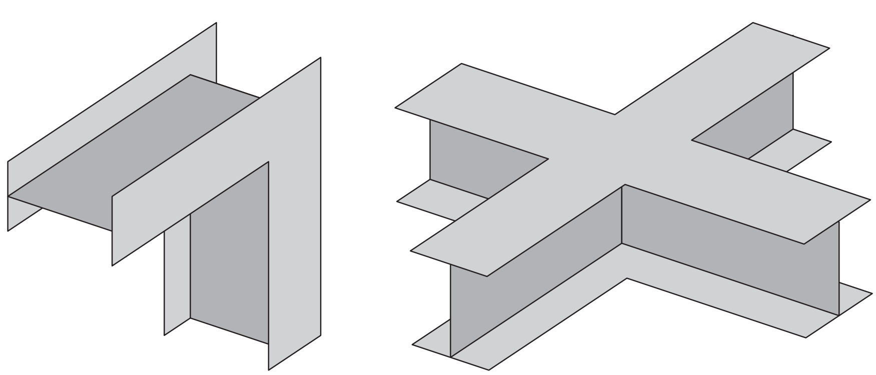

Fig. 1. Proper joining of thin-walled beams.