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Free vibration of quadrilateral laminated plates with carbon nanotube reinforced composite layers

P. Malekzadeh*, A.R. Zarei

Department of Mechanical Engineering, School of Engineering, Persian Gulf University, Bushehr 7516913798, Iran

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ABSTRACT

The free vibration behavior of quadrilateral laminated thin-to-moderately thick plates with carbon nanotube reinforced composite (CNTRC) layers is studied. The governing equations are based on the first-order shear deformation theory (FSDT). The solution procedure is based on transforming the governing differential equations from an arbitrary straight-sided physical domain to a regular computational one, and discretization of the spatial derivatives by employing the differential quadrature method (DQM) as an efficient and accurate numerical tool. Four different profiles of single walled carbon nanotubes (SWCNTs) distribution through the thickness of layers are considered, which are uniformly distributed (UD) and three others are functionally graded (FG) distributions. The fast rate of convergence of the presented approach is numerically demonstrated and to show its high accuracy, wherever possible comparison studies with the available results in the open literature are performed. Then, the effects of volume fraction of carbon nanotubes (CNTs), geometrical shape parameters, thickness-to-length and aspect ratios, different kinds of CNTs distribution along the layers thickness and different boundary conditions on the natural frequencies of laminated plates are studied.

1. Introduction

Due to high strength and stiffness-to-weight ratio of the carbon nanotube-reinforced polymer composites [1–8], the laminated plates of various shapes composed of these advanced composite materials have found wide applications as structural members in modern industries such as mechanical and aerospace engineering. Hence, the accurate evaluation of their vibration characteristics becomes essential for engineering design and manufacture.

The traditional approach to fabricate nanocomposites implies that the nanotubes are distributed either uniformly or randomly through the matrix such that the resulting material prosperities do not vary spatially at macroscopic level. However, both the experimental and numerical studies concerning carbon nanotube reinforced composites have shown that uniform distribution of CNTs as the reinforcements in the matrix can only achieve moderate improvement of the mechanical properties [1,4]. In addition, since these advanced composite material structures contain a low percentage of CNTs (2–5% by weight) [2,3,5], to effectively make use of the CNTs, Shen [9] suggested the use of a graded distribution of CNTs in the matrix. He showed that the nonlinear bending behavior of CNTRC plates can be considerably improved through the use of a FG distribution of CNTs in the matrix [9].

The vibration behaviors of rectangular single CNTRC layer plates and also sandwich plates with CNTRC face sheets have been investigated by some researchers in recent years, which are briefly reviewed.

Formica et al. [10] presented an equivalent continuum model according to the Mori–Tanaka scheme to study the vibration behavior of CNTRC rectangular plates. The investigated composite plates were made of a purely isotropic elastic hosting matrix of three different types (epoxy, rubber, and concrete) with embedded single-walled CNTs and the finite element method (FEM) was employed to solve the problem.

Thermo-mechanical nonlinear dynamic characteristics of rectangular CNTRC plates with uniformly distributed SWCNTs were studied by Wang and Shen [11]. They also analyzed the nonlinear vibration and bending of sandwich plates with functionally graded nanotube-reinforced composite face sheets [12]. In another work, they investigated the nonlinear dynamic response of carbon nanotube-reinforced composite (CNTRC) plates resting on elastic foundations in thermal environments [13]. Two configurations, i.e., single-layer CNTRC plate and three-layer plate that were composed of a homogeneous core layer and two CNTRC surface sheets, were considered. These studies were performed based on

* Corresponding author. Tel.: +98 771 4222150; fax: +98 771 4540376.
E-mail addresses: p_malekz@yahoo.com, malekzadeh@pgu.ac.ir (P. Malekzadeh).

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Reddy's higher-order shear deformation theory (HSDT) and a perturbation technique together with the Glerkin method was employed to solve the governing equations.

Zhu et al. [14] carried out the static and free vibration analysis of rectangular CNTRC plates based on the FSDT using the FEM. Four different distributions of CNTs through the thickness were considered. Lei et al. [15] employed the element-free kp-Ritz method to investigate the free vibration of rectangular functionally graded nanocomposite plates reinforced by SWCNTs. Natarajan et al. [16] studied the bending and free flexural vibration behavior of rectangular sandwich plates with CNTRC reinforced face sheets using a shear flexible finite element method developed based on the higher-order shear deformation theory. Nami and Janghorban [17] presented the three-dimensional free vibration of rectangular thick functionally graded carbon nanotube reinforced composite (FG-CNTRC) plates using the differential quadrature method.

To the best of authors' knowledge, the vibration characteristics of the quadrilateral laminated plates with CNTRC layers are not investigated yet. Hence, in this paper, the free vibration analyses of these types of composite structural elements under arbitrary boundary conditions are investigated. The formulation is based on the FSDT and is discretized using the DQM. The fast rate of convergence of the method is demonstrated and the results are validated by performing comparison studies with those obtained by using the commercial finite element package ANSYS and some other sources in the limit cases. The parametric studies are performed to exhibit the influences of volume fraction and orientation angle of CNTs, boundary conditions, width-to-thickness ratio and geometrical shape parameters of the quadrilateral plates on the natural frequencies. Layers with uniformly and graded distributed SWCNTs through their thickness are considered.

2. Theoretical formulation

Consider an arbitrary straight-sided quadrilateral laminated plate composed of perfectly bonded CNTRCs layers (Fig. 1). The physical coordinate system with coordinate variables (x, y, z) is

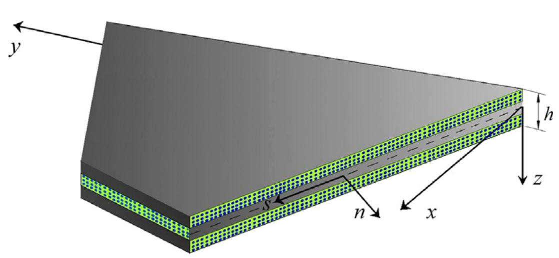

Fig. 1. The geometry of an arbitrary straight-sided quadrilateral laminated plate with CNTRC layers.

used to label the material points of the CNTRCs plates in the undeformed reference configuration. In the following, the related governing equations for free vibration analysis are stated.

2.1. Effective material properties of CNTRC layers

The SWCNTs are assumed to be aligned and straight. The material properties of SWCNTs are determined according to molecular dynamics (MD) simulations and then the effective material properties of CNTRCs are estimated through the rule of mixture in which the CNTs efficiency parameters are introduced to account for the scale-dependence of the resulting nanostructures. It is assumed that in each layer of the laminated plate, the distribution of CNTs is graded along the thickness direction as shown in Fig. 2. In order to model the influences of the CNTs on the overall properties of the composite plates, the extended rule of mixture as a simple and convenient micromechanics model [9,11–16,18] is used to obtain the effective material properties of the layers. According to it, the effective Young's modulus and shear modulus of a CNTRC layer can be expressed as [9]

$$E_{11}=\eta_1 V_{CN}E_{11}^{CN}+V_M E^M,\quad \frac{\eta_2}{E_{22}}=\frac{V_{CN}}{E_{22}^{CN}}+\frac{V_M}{E^M},\quad \frac{\eta_3}{G_{12}}=\frac{V_{CN}}{G_{12}^{CN}}+\frac{V_M}{G^M} \tag{1a–c}$$

where E_{11}^{CN}, E_{22}^{CN} and G_{12}^{CN} are Young's and shear moduli of the CNTs, E^M and G^M are the corresponding properties for the matrix, and the η_j ($j=1$, 2, 3) are the CNT efficiency parameters. In addition, V_{CN} and V_M are the volume fractions of the CNTs and the matrix in the layer respectively, which satisfy the relationship $V_{CN}+V_M=1$.

Various types of material profiles through the layer thickness can be considered. In this work, only linear distribution of the single walled CNT volume fraction for the different types of the CNTRC layer that can readily be achieved in practice is considered, which for a typical layer with the thickness $\bar{h}$ is as follows (Fig. 2):

$$\text{FG}-O:\quad V_{CNT}(z)=2\left(1-\frac{2|Z|}{\bar{h}}\right)V^*_{CNT}, \tag{2}$$

$$\text{FG}-V:\quad V_{CN}=\left(\frac{2Z+\bar{h}}{\bar{h}}\right)V^*_{CN}, \tag{3}$$

$$\text{FG}-X:\quad V_{CN}=4\left(\frac{|Z|}{\bar{h}}\right)V^*_{CN} \tag{4}$$

where

$$V^*_{CN}=\frac{w_{CN}}{w_{CN}+\left(\frac{\rho^{CN}}{\rho^M}\right)-\left(\frac{\rho^{CN}}{\rho^M}\right)w_{CN}} \tag{5}$$

w_{CN} is the mass fraction of nanotube, and ρ^{CN} and ρ^M are the densities of carbon nanotube and matrix, respectively; Z is a local thickness coordinate variable for a typical layer. Note that

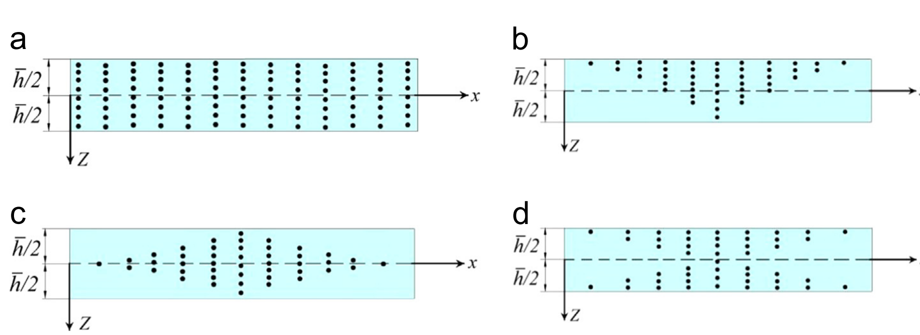

Fig. 2. Different kinds of distributions of CNTs along the thickness of a typical layer: (a) uniform distribution (UD); (b) functionally graded (FG-V); functionally graded (FG-O) and functionally graded (FG-X).