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Finite element method for stability and free vibration analyses of non-prismatic thin-walled beams

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ABSTRACT

In this paper, a numerical method is presented for the free vibration and stability analyses of tapered thin-walled beams with arbitrary open cross sections. The proposed method takes the flexural–torsional coupling effect of tapered thin-walled beams with arbitrary open cross sections into account. The total potential energy is derived for an elastic behavior from the strain energy, the kinetic energy and work of the loads applied on the cross section contour. Free vibration is considered in the presence of harmonic excitations. The effects of the initial stresses and load eccentricities are also considered in stability analysis. The governing equilibrium equations, motion equations and the associated boundary conditions are derived from the stationary condition. As in the presence of tapering, stiffness quantities are not constant; therefore, the power series approximation is used to solve the fourth-order differential equations. Displacement components and cross-section properties are expanded in terms of power series of a known degree. Then, the shape functions are obtained by deriving the deformation shape of tapered thin-walled member as power series form. Finally, stiffness and mass matrices are carried out by means of the principle of virtual work along the member's axis. In order to measure the accuracy and check the validity of this method, the natural frequencies and buckling loads of non-prismatic thin-walled beams with web and flange tapering and various boundary conditions are obtained and compared to the results of finite element analysis using Ansys software and those of other available numerical and analytical ones. It can be seen that the results of present study are in a good agreement with other available theoretical and analytical methods.

1. Introduction

The use of thin walled beams with open variable cross sections has been increasing in steel structures due to their ability to utilize structural material more efficiently and optimize the distribution of weight. With the development of manufacturing process, tapered beams are now adopted with other materials such as wood and concrete. However, prediction of the flexural–torsional buckling loads and vibration of thin walled beams with arbitrary cross-section is complex because of coupling of torsion and bending. The task seems to be more complicated in the presence of non-prismatic thin walled beams where the cross section properties are not constant. The accurate prediction of the buckling load and natural frequencies is one of the important points in the design of thin-walled structures. Meanwhile correct evaluation of stability and free vibration of non-prismatic thin-walled beam with non-symmetric cross-section are of interest to many researchers.

The Timoshenko and Vlasov models [1,2] constitute a powerful background of the rich extensive literature on the elastic flexural–torsional buckling behavior of thin walled beams. Chen, Liu and Bazant, Cedolin evaluated buckling loads either by closed-form solutions of the fourth-order differential equations governing the twisting and bending of the thin-walled beams or by using of complementary energy principle [3,4]. However, these studies are mainly focused on the stability analysis of uniform cross-section beams. Different numerical methods as finite element methods have been developed by many researchers in the case of stability and vibration analyses of prismatic thin walled beams with mono symmetric or arbitrary cross section shapes [5–21]. Most of these studies have been concerned on the stability and vibration analyses of thin-walled beams with uniform cross sections. However, in the last decades, the thin-walled with variable cross sections have been used in different steel constructions; therefore, studying their vibration and stability behavior has gained more attention by many authors [22–36]. The investigations of elastic flexural, lateral-torsional buckling load and natural frequency of tapered thin-walled with the recourse to finite element method in the validation process has been attracted by many researchers.

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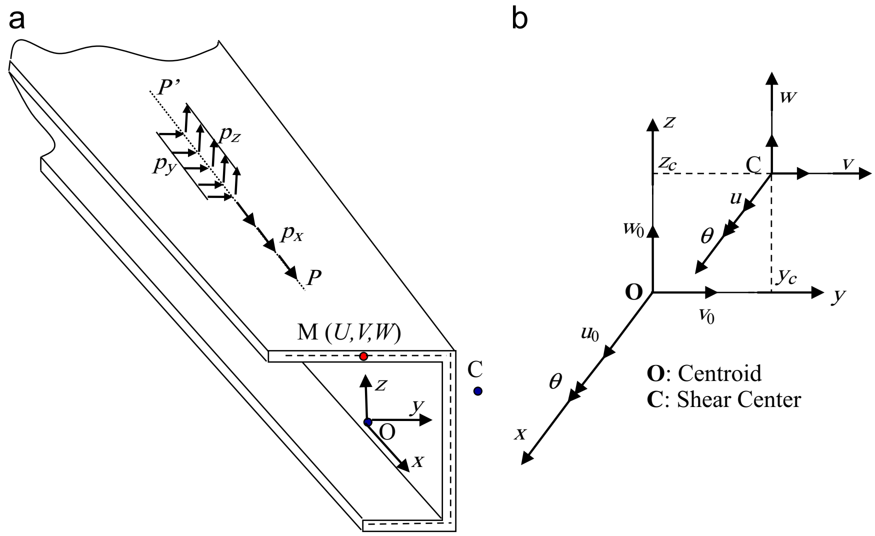

Fig. 1. (a) Thin-walled beam with non-symmetric cross-section and (b) coordinate system and notation for displacement parameters of the non-prismatic beam.

Among the first investigations on this topic, the most important ones are the study of Brown [22] who used shell elements in mesh process of tapered I beams and carried out their buckling loads accordingly. This work was followed by the studies of Yang [23] who used the updated Lagrangian approach to formulate a finite element model for geometrical non-linear analysis. Pasquino [24] used a variational approach to derive the Euler–Lagrange equations to evaluate the buckling loads of non-prismatic thin-walled beams with arbitrary cross-section. A 3D beam finite element model for large torsion context applied to the case of doubly symmetric tapered cross-sections was developed by Ronagh [25,26] in which effects of non-linear terms such as flexural–torsional coupling in the presence of cubic terms of torsion angle were considered. Chen [27] obtained the motion equations governing of non-prismatic thin-walled beams with arbitrary cross-sections by using Hamilton's principle. A finite element approach was introduced by Kim [28] to investigate the linear stability and free vibration behavior of doubly symmetric I tapered thin-walled beam. Li [29] used the Chebyshev polynomial method to solve the equilibrium differential equation of tapered beams under constant axial load and by considering the effect of shear deformation.

The elastic and geometric stiffness matrices of the non-uniform beam element with I section subjected to torsional load is determined by Yau [30]. In the mentioned study, the effects of the non-uniform torsions and the second-order of warping torsions are pondered. Lei [31] established beam strain energy and equilibrium equations considering deformation compatibilities of two the flanges and web of the I-section and investigated the linear stability behavior of the tapered beams. Improved solutions were obtained. More recently, based on the Rayleigh–Ritz method combined with shell element, a general variational formulation to analyze the lateral-torsional buckling behavior of simply supported and cantilever tapered thin-walled beams with singly symmetric cross-sections was presented by Andrade [32–34]. Effects of load position were also modeled in these studies. The lateral stability of thin-walled tapered beams with singly symmetric I cross-sections by means of the power series method was investigated by Asgarian [35]. The previous method has been extended to stability and free vibration behavior of tapered beams with non-symmetric cross-sections and arbitrary boundary conditions using power series expansions by Soltani [36]. Most of the previous works devoted to tapered beams have been limited to doubly or singly symmetric cross-sections in both stability and vibration analyses.

In the present paper, power series method is combined with a finite element approach and applied to tapered beams with arbitrary cross sections. After discretization steps, the stiffness and mass matrices needed in stability and free vibration analyses are determined for non-prismatic thin-walled beams subjected to constant eccentric axial and variable bending loads. The contents of this study are summarized as follows:

1. The equations of motion are derived from the energy principle of the thin-walled beam subjected to bending and axial loads. Load positions on the cross section contour are also taken into account.
2. The power series expansions are used to facilitate the solution of the fourth-order differential equation of equilibrium of non-prismatic thin-walled member with variable coefficients. In this regard, it is assumed that the functions which describe the beam's variable parameters such as: flexural and warping rigidities, density and loads are expanded into power series form. The expressions of shape function are also determined based on aforementioned method.
3. The terms of stiffness and mass matrices are derived by means of the shape functions resulting from its nodal displacements and the principle of virtual work along the beam axis. The critical buckling loads and natural frequencies are also obtained by solving the eigenvalue problem.

After presenting above items, several numerical examples are presented in order to measure the accuracy and validity of the proposed method. The results are also compared with other available results. The present method has many advantages including efficiency, accuracy and simplicity compared to more complicated numerical methods. The main advantage of this method is its simplicities in which for stability and vibration analyses of structure, only geometry and material properties of the members need to be defined.

2. Derivation of equilibrium and motion equations for buckling and free vibration

2.1. Kinematic

A non-prismatic thin-walled beam with arbitrary cross-section as shown in Fig. 1a is considered. It is assumed that the beam is made from homogenous and isotropic material and the length of