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Review

Semi-analytical models for the post-buckling analysis and ultimate strength prediction of isotropic and orthotropic plates under uniaxial compression with the unloaded edges free from stresses

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ABSTRACT

This paper introduces two semi-analytical models developed for the nonlinear analysis of stability of isotropic and orthotropic plates under uniaxial compression. The possibility of considering fully free in-plane displacements at longitudinal edges (or unloaded edges) is the innovation of these models over existing models, where these displacements are always assumed constrained to remain straight. Contributions for the large deflection theory of plates related to the derivation of analytical solutions for the Airy stress function which satisfy Marguerre's equations for isotropic and orthotropic plates are presented. Namely, the extension of the Coan and Urbana solution for isotropic plates in order to consider all the terms of the unknown amplitudes of the out-of-plane displacements and the derivation of a solution for orthotropic plates. Comparisons between the semi-analytical model and nonlinear finite element model results are presented in order to discuss the effect of in-plane displacement boundary conditions on behaviour and strength of plates similar to bottom flanges used in steel box girder bridges. This study shows that the semi-analytical models have a clear potential to provide accurate solutions, requiring only a short computer time. It is also shown that the in-plane displacement boundary conditions for the unloaded edges significantly influence the behaviour and strength of plates and this problem cannot be neglected in the definition of the design rules.

Contents

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1. Introduction

The nonlinear finite element simulation used for the analysis and design of steel plated structures is a powerful tool especially for research, but for the design purpose it is neither generalised nor competitive. Recent work in the field of semi-analytical methods used for the nonlinear analysis and design of steel plated structures with buckling problems has shown an important and alternative tool that provides an efficient and understandable response [1–9].

The semi-analytical method uses the two nonlinear fourth-order partial differential equations of the large deflection theory, the equilibrium and compatibility equations derived by von Kármán in 1910 [10] for perfect plates and extended to plates with initial imperfections by Marguerre [11]. The method is called semi-analytical because in a first step an analytical solution for the Airy stress function (F) is obtained solving the compatibility equation. Trigonometric series satisfying the boundary conditions are adopted for the out-of-plane displacements (w) and for the initial imperfections (w_0). In a second step approximate solutions for the unknown amplitudes (q) of the out-of-plane displacements are obtained solving the equilibrium equation using a variational method. Based on the approximate solutions for the unknown amplitudes of the out-of-plane displacements, on the kinematic and constitutive relations and on the yield criterion it is possible to analyse the post-buckling behaviour and to predict the ultimate strength of the plate.

Analyses of the post-buckling behaviour of isotropic and orthotropic plates under uniaxial compression with all edges simply supported were reported by several authors since the early nineteen forties. These studies are well documented in reference works [12,13], from which can be highlighted the analytical solutions for the Airy stress function, which satisfy von Kármán's equations for plates with the in-plane displacements perpendicular to the edges constrained to remain straight in all edges, derived by Levy [14] for isotropic plates and by Soper [15] for orthotropic plates and the analytical solution for the Airy stress function that satisfies Marguerre's equations for isotropic plates with in-plane displacements perpendicular to the edges constrained to remain straight in loaded edges and free in unloaded edges, derived by Coan and Urbana [16]. The Coan and Urbana study was limited to odd number terms m and n for the unknown amplitudes (q_{mn}) of the out-of-plane displacements and it was extended by Yamaki [17] to include the even number terms n, where m and n represent the number of half-waves of the out-of-plane displacement mode in the longitudinal and transverse directions respectively.

Nowadays two computational programs use the semi-analytical method to analyse the post-buckling behaviour of plates and to predict their ultimate strength. The computer program ALPS/ULSAP, which uses a semi-analytical method previously known as the incremental Galerkin method [18] and the computer program PULS developed at Det Norske Veritas (DNV) and accepted as general buckling code for ship and offshore platform structures as part of the DNV specifications [19]. The main difference between the two programs is how the equilibrium equation is solved. The ALPS/ULSAP program uses the Galerkin method [20,21], while the PULS program uses the Rayleigh–Ritz method [1,22].

The semi-analytical models which have been developed are restricted to plates supported by rigid transverse and longitudinal girders. This kind of arrangement is typical in ship, aircraft, tank and offshore platform structures where it is assumed that the plate is simply supported and the in-plane displacements perpendicular to the edges are constrained to remain straight in all edges. Generally, in bottom flanges of steel box girder bridges there are no neighbouring panels in the longitudinal edges to provide this kind of constraint and it is on the safe side to consider the longitudinal edges with fully free in-plane displacements that are characteristic of edges free from stresses.

This paper introduces two semi-analytical models developed for the nonlinear analysis of stability of isotropic and orthotropic plates under uniaxial compression that allow us to consider the two types of in-plane displacement boundary conditions mentioned above: in-plane displacements perpendicular to the edges constrained to remain straight in all edges (designated as case *CC*) and in-plane displacements perpendicular to the edges constrained to remain straight in loaded edges and free in unloaded edges (designated as case *CF*). The case *CF* for the in-plane displacement boundary conditions is the innovation of the presented models over existing ones.

In this study the semi-analytical model for isotropic plates is used for the analysis of unstiffened plates used in steel girder bridges and the semi-analytical model for orthotropic plates is used for the analysis of stiffened plates. The stiffened plate is treated in an approximate way as an equivalent orthotropic plate.

In order to develop these semi-analytical models analytical solutions should be obtained in the scope of the large deflection theory of plates considering the case *CF* for the in-plane displacement boundary conditions, namely to extend the Coan and Urbana [16] solution for isotropic plates to include the even and odd number terms m and n for the unknown amplitudes of the out-of-plane displacements and to derive a solution for orthotropic plates. This paper presents analytical solutions for the Airy stress function satisfying Marguerre's equations for isotropic and orthotropic plates to solve these two issues, which are contributions of this work for the large deflection theory of plates.

Finally, the paper presents a comparison between the semi-analytical model and nonlinear finite element model results. The nonlinear finite element model results were obtained using the program ADINA [23]. The presented results also allow a discussion about the effect of the in-plane displacement boundary conditions for the unloaded edges on the behaviour and strength of plates.

2. Analysis method

2.1. General

The coordinate system and the notation for the theoretical analysis presented in this paper are shown in Fig. 1. A simply supported plate with an initial imperfection w_0, width b, length a and thickness t under longitudinal uniform compression σ is considered, in which the xy plane coincides with the plate mid-surface and the z-axis is perpendicular to the plate mid-surface. The displacement components at a point, occurring in the x, y and z directions, are denoted by u, v and w respectively.

In order to study the effect of in-plane displacement boundary conditions for the unloaded edges on the behaviour and strength