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Nonlinear vibration of functionally graded cylindrical shells embedded with a piezoelectric layer

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ABSTRACT

This paper addresses the nonlinear vibration problem of simply supported functionally graded (FG) cylindrical shells with embedded piezoelectric layers. The governing differential equations of motion of the FG cylindrical shell are derived using the Lagrange equations under the assumption of the Donnell's nonlinear shallow-shell theory. A semi analytical approach, wherein the displacement fields are expanded by means of a double mixed series based on linear mode shape functions for the longitudinal, circumferential and radial variables, is proposed to characterize the nonlinear response of the cylindrical shell. The large-amplitude response and amplitude frequency curves of the cylindrical shell are obtained by using the proposed approach. Finally, the effects of excitation force and applied voltage on the vibration behavior of the cylindrical shell are investigated.

1. Introduction

Functionally graded materials (FGMs) are a class of composite materials consisting of two or more different materials whose material properties are gradually varied in one or more directions. Due to this favorable feature, such type of composites provides the possibility to develop materials whose properties can be precisely adjusted to the requirements of a certain application. For this reason, FGMs have found many applications in different branches of engineering such as nuclear, mechanical, aerospace, and automotive engineering [1–6].

On the other hand, the FGMs can be integrated with piezoelectric materials in order to improve their dynamical behaviors. The piezoelectric materials are well-known for their sensing and actuating capabilities. These structures have the ability to control the size, shape, vibration and stability of the structural components due to their direct and converse piezoelectric effects. For example, a piezoelectric sensor layer can monitor the deformation of a structure while a piezoelectric actuator layer can control the deformation of the structure through the converse piezoelectric effect. For this reason, the piezoelectric materials have found many applications in vibration control and monitoring [7–10]. Therefore, it is of great importance to analyze the behavior of structures made of FGMs integrated with piezoelectric materials.

Due to above-mentioned favorable features, the analysis of linear and nonlinear vibration response of FGMs and piezoelectric materials has attracted research interest in recent years. An analytical study on nonlinear dynamic stability of simply supported circular cylindrical shells composed of FGM under periodic axial loading was performed by Darabi et al. [11]. Nonlinear buckling and postbuckling behaviors of FG cylindrical shells which are synchronously subjected to axial compression and lateral loads were studied by Huang and Han [12]. Nonlinear thermoelasticity, vibration, and stress wave propagation analysis of thick-walled cylinders made of FGMs with temperature-dependent properties was performed by Shariyat et al. [13]. Duc and Tung [14] proposed an analytical approach to investigate the nonlinear response of FG cylindrical panels under uniform lateral pressure with temperature effects is incorporated. The non-linear free vibration of a FG doubly-curved shallow shell of elliptical plan-form was investigated by Chorfi and Houmat [15] using the p-version of the finite element method in conjunction with the blending function method. The effects of transverse shear deformations, rotary inertia, and geometrical non-linearity were also studied. An attempt on the dynamic control of FGM shells in the frequency domain was carried out by Liew et al. [16] by using self-monitoring sensors and self-controlling actuators. The coupled vibration of an inhomogeneous orthotropic piezoelectric hollow cylinder filled with internal compressible fluid was studied by Chen et al. [17]. The analysis was directly based on the three-dimensional equations of piezoelasticity and the cylinder shell was assumed to have a FG property along the thickness direction. Three-dimensional static behavior of doubly curved FG magneto-electro-elastic shells under the mechanical load, electric displacement and magnetic flux was studied by Wu and Tsai [18] by an asymptotic approach. An analytical study for electromagnetothermoelastic behaviors of a hollow cylinder composed of functionally

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graded piezoelectric material (FGPM) placed in a uniform magnetic field and subjected to electric, thermal and mechanical loadings was carried out by Dai et al. [19]. The free vibration problem of multi-layered shells with embedded piezoelectric layers was investigated by Alibeigloo and Kani [20] using a hybrid state space differential quadrature method. However, to the authors' best knowledge, the information regarding the nonlinear vibration of FG cylindrical shells with embedded piezoelectric layers is rare and this is the reason why this paper tries to investigate this topic in the paper.

In the present work, the linear and nonlinear vibrations of simply supported FG cylindrical shells with piezoelectric layer on their outer surface are studied. The properties of FGM are assumed to be graded in the thickness direction according to a volume fraction power law distribution. The formulation of the problem is based on the Donnell's nonlinear shallow-shell theory and the Fourier series expansion method is used to solve the problem. In the case of linear analysis, the fundamental frequencies and mode shapes of the cylindrical shell are calculated. And, in the case of nonlinear analysis, the governing equations are derived using the Lagrange equations and solved using the ODE45 Runge–Kutta routine in MATLAB. The time–amplitude response and amplitude frequency curves of the cylindrical shell are obtained and the effects of excitation force and applied external voltage on vibration characteristics of the cylindrical shell are examined.

2. Governing equations and solution procedure

2.1. Linear analysis

Fig. 1 shows the coordinate system of a FG cylindrical shell which is embedded with a piezoelectric material in its outer surface. The geometrical parameters of the cylindrical shell are mid-surface radius (R), thickness of FG cylindrical shell (h), thickness of piezoelectric layer (h_p) and length (L). The cylindrical shell is assumed to be thin with a uniform thickness ($h+h_p$). The displacement components in the x, θ and z directions are denoted by u, v and w, respectively.

As pointed out earlier, FGMs are composite materials obtained by combining and mixing two or more different constituent materials. In this paper the FG material is assumed to be combined form two constituent materials. The Young modulus E, Poisson ratio ν, and the mass density ρ of the FG cylindrical shell are assumed to vary through the thickness according to the power-law function as follows [21]:

$$E_F=(E_T-E_B)\left(\frac{2z+h}{2h}\right)^N+E_B \quad (1)$$

$$\nu_F=(\nu_T-\nu_B)\left(\frac{2z+h}{2h}\right)^N+\nu_B \quad (2)$$

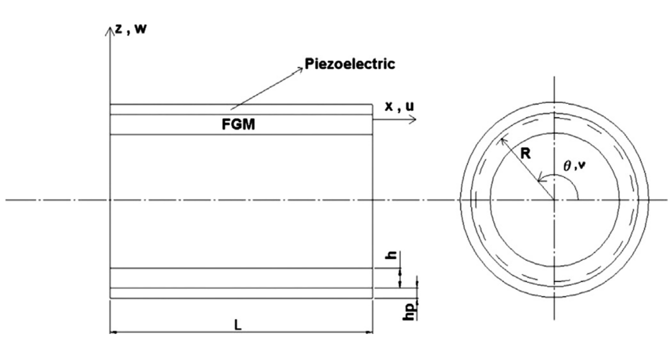

Fig. 1. Coordinate system of FG cylindrical shell with piezoelectric layer.

$$\rho_F=(\rho_T-\rho_B)\left(\frac{2z+h}{2h}\right)^N+\rho_B \quad (3)$$

where N is the power-law exponent. Furthermore, the subscripts T and B indicate the properties of the FG cylindrical shell at its top and bottom surfaces, respectively.

Based on the state of generalized plane stress of shells, the normal stress is assumed to be zero in the radial direction. In this regard, the fundamental equations of the FG and piezoelectric materials for a thin cylindrical shell can be expressed as [22]:

$$\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_{x\theta}\end{Bmatrix}=\begin{bmatrix}Q_{11F}&Q_{12F}&0\\ Q_{12F}&Q_{22F}&0\\ 0&0&Q_{66F}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_{x\theta}\end{Bmatrix}\quad \text{or}\quad \{\sigma_F\}=[Q_F]\{\varepsilon\} \quad (4)$$

for FGM, and

$$\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_{x\theta}\end{Bmatrix}=\begin{bmatrix}Q_{11P}&Q_{12P}&0\\ Q_{12P}&Q_{22P}&0\\ 0&0&Q_{66P}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_{x\theta}\end{Bmatrix}-\begin{bmatrix}0&0&e_{31}\\ 0&0&e_{32}\\ 0&0&0\end{bmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix}$$

$$\text{or}\quad \{\sigma_P\}=[Q_P]\{\varepsilon\}-[e]\{E\} \quad (5)$$

$$\begin{Bmatrix}D_x\\ D_\theta\\ D_z\end{Bmatrix}=\begin{bmatrix}0&0&0\\ 0&0&0\\ e_{31}&e_{32}&0\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_{x\theta}\end{Bmatrix}+\begin{bmatrix}\xi_{11}&0&0\\ 0&\xi_{22}&0\\ 0&0&\xi_{33}\end{bmatrix}\begin{Bmatrix}E_x\\ E_\theta\\ E_z\end{Bmatrix}$$

$$\text{or}\quad \{D_P\}=[e]^T\{\varepsilon\}+[\xi]\{E\} \quad (6)$$

for piezoelectric material, where the subscripts F and P denotes FGM and piezoelectric material, respectively; $\{\sigma\}$, $\{\varepsilon\}$, $\{D\}$ and $\{E\}$ are the vectors of stress, strain, electric induction, and the electric field, respectively; $[Q_F]$, $[Q_P]$, $[e]$ and $[\xi]$ denote the elastic constants for FGM, elastic constants for piezoelectric material, piezoelectric constants and dielectric constants, respectively. The elastic constants, for both FGM and piezoelectric materials, are defined as follows:

$$Q_{11F}=Q_{22F}=\frac{E_F}{1-\nu_F^2}\quad Q_{12F}=\frac{\nu_F E_F}{1-\nu_F^2}\quad Q_{66F}=\frac{E_F}{2(1+\nu_F)} \quad (7)$$

$$Q_{11P}=Q_{22P}=\frac{E_P}{1-\nu_p^2}\quad Q_{12P}=\frac{\nu_P E_P}{1-\nu_p^2}\quad Q_{66p}=\frac{E_p}{2(1+\nu_P)} \quad (8)$$

The components of the electric field (i.e., E_x, E_θ, and E_z) are defined by the electric potential function ϕ in the curvilinear coordinate system as follows [23]:

$$E_x=-\frac{\partial\phi}{\partial x},\quad E_\theta=-\frac{1}{R}\frac{\partial\phi}{\partial\theta},\quad E_z=-\frac{\partial\phi}{\partial z} \quad (9)$$

For considering both the direct and converse piezoelectric effects, the electric potential function ϕ is defined as a layerwise quadratic distribution of the electric potential according to the Fernandes and Pouget model [24]:

$$\phi=2\frac{G\left(z-\frac{h+hp}{2}\right)}{hp}+\left(\left(z-\frac{h+hp}{2}\right)^2-\frac{hp^2}{4}\right)\psi(x,\theta,t) \quad (10)$$

where $\psi(x,\ \theta,\ t)$ is the induced electric potential by elastic deformation in the piezoelectric element, and G is the electric potential applied on the piezoelectric surfaces such that:

$$\phi(z=h/2+hp)=G \quad (11)$$

$$\phi(z=h/2)=-G \quad (12)$$

The strain components ε_x, ε_θ and $\varepsilon_{x\theta}$ which are the strains in the x-direction, the circumferential direction and the shear strain in the $x\theta$-plane of the middle surface, respectively, can be expressed as: