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Vibration correlation technique for the estimation of real boundary conditions and buckling load of unstiffened plates and cylindrical shells

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ABSTRACT

Nondestructive experimental methods to calculate the buckling load of imperfection sensitive thin-walled structures are one of the most important techniques for the validation of new structures and numerical models of large scale aerospace structures. Vibration correlation technique (VCT) allows determining equivalent boundary conditions and buckling load for several types of structures without reaching the instability point. VCT is already widely used for beam structures, but the technique is still under development for thin-walled plates and shells. This paper intends to explain the capabilities and current limitations of this technique applied to two types of structures under buckling conditions: flat plates and cylindrical shells prone to buckling. Experimental results for a flat plate and a cylindrical shell are presented together with reliable finite element models for both cases. Preliminary results showed that the VCT can be used to determine the realistic boundary conditions of a given test setup, providing valuable data for the estimation of the buckling load by finite element models. Also numerical results herein presented show that VCT can be used as a nondestructive tool to estimate the buckling load of unstiffened cylindrical shells. Experimental tests are currently under development to further validate the approach proposed herein.

1. Introduction

Nondestructive methods for buckling prediction were initially developed for very simple cases like columns, e.g. the Southwell method presented in the early 1930s [1]. Vibration correlation methods, which originated from the similarities found between the buckling and vibration modes for columns and another structures, can also be used as a nondestructive technique. The concept of relating vibration characteristics to buckling loads was considered at the beginning of the 20th century for Somerfeld [2], but only in the 50's some experimental investigations were conducted by Chu [3]; Lurie [4] and Meier [5], among others.

It can be easily shown ([6]) that the relationship between the squared frequency and the compressive load is exactly linear in the case of simple supports columns

$$\left(\frac{f_n}{f_{n0}}\right)^2 = 1 - \left(\frac{P}{P_n}\right) \quad (1)$$

where P_n is the buckling load, f_{n0} is the nth frequency of the unload column, and f_n is the nth frequency of the column with an applied P load.

Furthermore, Massonnet [7] and Lurie [4] also investigated columns with different boundary conditions showing that the relation between the buckling load and the squared frequency were nearly linear related for all cases.

Despite the use of vibration correlation technique (VCT) for columns has been found practical as a nondestructive method for calculating buckling load, their application for plates and shells showed to be not as straightforward. Lurie [4] extended the concept of VCT for plates and found the same Eq. (1) as an exact solution for thin plates of polygonal shape, uniform thickness and

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simply supported in all edges. For perfect plates with other different shapes or other boundary conditions some small deviation should be expected. However, initial geometric imperfections in plates have a significant effect on the buckling load and may cause considerable deviations from linearity with the squared frequency. Lurie [4] presented some experimental results with geometrical imperfect plates and showed a remarkable deviation from linearity. Later on, Chailleux et al. [8] performed more careful test in plates with smaller imperfections and confirmed Lurie's findings: VCT is reliable only when applied in specimens which have small initial imperfections.

Jubb et al. [9] carried-out another series of test applying VCT to a box-column made of four thick plates welded together. Several vibration modes along the plates were measured when the structure was loaded under axial compression. It was noticed that the vibration mode with a shape similar to the buckling mode provided the best linear fit between the buckling load and the squared frequency. Similar linear fits using other vibration modes led to higher buckling loads compared to experimental results. Moreover, Jubb et al. [9] observed that the vibration frequencies started to increase after buckling. This slope change, or the lowest point of the frequency, can be employed as a criterion to characterize the buckling load. In this context, several tests were carried out by Abramovich et al. [10], with radially loaded circular plates, obtaining a good correlation between the experimental buckling load and the minimum frequency value. On the other hand, Abramovich et al. [10] found that the experimental buckling load was 15% below the theoretical value for perfect plates.

Ilanko et al. [11,12] performed experimental test in simply supported plates loaded under axial compression up to the post-buckling regime. Their conclusions were in agreement with Jubb's findings [9].

Despite the VCT have been successfully applied to columns and plates, their capabilities are not fully exploited yet for shells. Several extensive studies were conducted at the Technion Aerospace Structural Laboratory in the 1970s with stiffened cylindrical shells. The similarities in the influence of boundary conditions on the buckling of stiffened shells and their free vibration behavior motivated the extensive studies at Technion to correlate these two phenomena and estimate from the nondestructive vibration results the destructive buckling behavior of these shells (see [13,14]).

For a better understanding of the applications of VCT on plates and shells it is important to classify the method according to its use: (1) for the determination of actual boundary conditions for numerical calculation purposes; and (2) for direct calculation of buckling load.

The VCT for the determination of boundary conditions consists essentially of an experimental determination of the lower natural frequencies for a loaded shell and evaluation of equivalent elastic restraints representing the actual boundary conditions. As an example, a compilation of test results carried out by Technion was presented by Singer [15]. The studies performed in 35 shells showed a significant reduction of the knock-down factor scatter as a result of including the experimental determination of the boundary conditions the prediction of the buckling load.

Technion studies were not limited only to laboratory boundary conditions. To establish a reliable methodology, VCT was applied to structures with realistic boundary conditions like component joints commonly used in aerospace, taking into account also the introduction of load eccentricity. Singer et al. [16] presented a detailed modified VCT to define the boundaries, once the load eccentricity has been identified, showing a good correlation with experimental results.

The VCT for the determination of boundary conditions has also been extended to the direct prediction of buckling loads, particularly for stiffened shells with their low frequency vibration modes observed in tests looking very similar to their buckling modes ([13]). For the direct prediction of buckling loads it is essential to perform a curve fitting with experimental points, but using only those points below 50% of the buckling load to make the method effectively nondestructive. For cylindrical stiffened shells the curve fitting used has the form

$$f^q = A - BP \tag{2}$$

where f is the frequency, P the applied load, A and B are fitting constants. The parameter q is an empirical value calculated from experimental data. Segal [17] conducted a parametric study to determine the q value for several integrally stringer-stiffened cylinders tested at the Technion under axial compression.

Based on Technion experimental tests Souza et al. [18] suggested a different straight-line technique than the one presented in Eq. (2). Instead of plotting the frequency to the appropriate power q versus P and fitting the best straight line, they proposed plotting $(1-p)^2$ versus $(1-f^4)$, where $p=(P/P_{cr})$, $f=(f_m/f_0)$, P is the applied axial load, P_{cr} is the critical buckling load for a perfect shell, f_m is the measured frequency at P load and f_0 is the natural frequency of the unloaded perfect shell. The value of $(1-p^2)$ corresponding to $(1-f^4)=1$ would represent the square of the drop of the load carrying capacity (ξ^2), due to the initial imperfections. The results showed a good agreement with Technion tests.

Moussaoui [19] presents a detailed literature review focused on the most relevant research for the study of non-linear vibration in structures such as plates and shells, where more details about previous work regarding the VCT can be found. The lecture of Singer et al. [6] (chapter 15th) should also be consulted for a deeper insight on this topic and a fully explanatory review about experimental setup and results.

There is no complete understanding of how to apply the VCT for unstiffened cylindrical shells, commonly used in space applications. A high imperfection sensitivity is usually associated with this type of structure, which requires the application of empirical guidelines in order to calculate the design buckling load, currently leading to conservative estimations ([20]). Skukis et al. [21] presented a preliminary assessment of correlation between vibration and buckling load of stainless steel cylinders. If a relationship between the buckling load and the variation of the natural frequencies of vibration exists, it is possible to use the VCT as a non-destructive technique for a better estimation of the knock-down factor for space structures. Moreover, for this type of structures there is a remarkable influence of the boundary conditions on the buckling load (see [22,23]), where the VCT could be used for a better characterization of the actual boundary conditions, providing reliable data for numerical simulation, such as finite element models (see [24,25,20]).

Focusing on these open questions, this paper shows the capabilities and limitations of the VCT applied to thin-walled structures and the advantages of the finite element modeling taking into account pre-existing information about the boundary conditions from the VCT and measured initial geometric imperfections, in order to calculate the buckling load.

A benchmark case of the VCT application based on an isotropic plate loaded under axial compression up to the post-buckling regime is presented in order to have reliable data for the next studies. In the sequence, a finite element model is developed to show the improvement on correlation when the VCT and initial geometric imperfections are taken into account.

Finally, preliminary tests using the VCT are conducted on a 300 mm diameter cylinder, fabricated with composite materials, in order to identify the range of applicability of the VCT for unstiffened cylindrical shells. Based on these observations, a new methodology to estimate the buckling load of unstiffened cylindrical shells using the VCT is proposed. Preliminary results presented through a finite element model showed a good correlation of the estimated buckling load.