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Dynamic stability of cantilevered functionally graded cylindrical shells under axial follower forces

Mohammad Ebrahim Torki [a], Mohammad Taghi Kazemi [b,1], Hassan Haddadpour [c,*], Saeed Mahmoudkhani [d]

[a] Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843, USA
[b] Department of Civil Engineering, Sharif University of Technology, postal code: 111558639 Tehran, Islamic Republic of Iran
[c] Department of Aerospace Engineering, Sharif University of Technology, postal code: 111558639 Tehran, Islamic Republic of Iran
[d] Department of Aero-Space Engineering, Sharif University of Technology, postal code: 111558639 Tehran, Islamic Republic of Iran

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ABSTRACT

Flutter of cantilevered, functionally graded cylindrical shells under an end axial follower force is addressed. The material properties are assumed to be graded along the thickness direction according to a simple power law. Using the Hamilton principle, the governing equations of motion are derived based on the first-order shear deformation theory. The stability analysis is carried out using the extended Galerkin method and minimum flutter loads and corresponding circumferential mode numbers are obtained for different volume fractions, length-to-radius, and thicknesses-to-radius ratios. Two different configurations are considered for the FGM: one in which the metal phase is the outer layer and the ceramic phase is the inner one, and the other vice versa. Results indicate the ranges of major influence due to the volume fraction, and the combined effect of thickness and volume fraction on the flutter load. Also, the optimum and critical power parameters between zero and infinity for which the flutter loads are maximum and minimum are determined.

1. Introduction

The flutter problem dates back to 1969 in cantilever rubber pipes containing low-pressure air flows [1]. Two basic types of instability may exist in pipes containing fluid flows: one characterized with zero frequency, called divergence, and the other with non-zero frequency, known as flutter. However, the prevailing instability type in these structures is flutter, which occurs in high-speed fluid flows. When undergoing follower forces, the only instability type in structures with moderate thicknesses is flutter, which is, for the most part, limited to columns, reservoirs, and aero-space structures [2]. The most well-known problem pertaining to follower forces is Beck's problem, in which a concentrated follower force is applied at the free end of a cantilever. Practical applications of this problem include the thrust applied on the end of a projectile, gas turbine rotors, the gripping force in disk brakes, the thrust applied on the body of aircraft structures by a jet engine, the eccentric load exerted on a platform by a tip mass, etc. [3–5].

In recent years, *Functionally-Graded Materials* (*FGM*) have received wide applications in engineering mechanics since laminated composites can encounter delamination when undergoing great mechanical or temperature loads due to different deformation fields occurring in different layers which lead to inter-layer stresses. Thus, in order to control the mechanical properties including the amounts and localities of temperature stresses, yielding and ultimate strengths, and crack stimuli and zones, *FG* materials are generally preferable to laminated composites [6]. Of the most notable applications of *FGM* is in air-plane landing gears, reservoirs containing chemical, radioactive, or plasma settings, high-speed aircrafts (including skin structures such as fuselages), propulsion systems in air planes, cutting instruments, incinerators, heat exchangers, turbine blades, etc. [7–12].

Some works about flutter of cylindrical shells under follower forces are reported in the literature. In this respect, Altman and De Oliviera [13,14] studied the dynamic stability of cantilever cylindrical and conical panels with and without slight internal damping. They asserted that due to numerical defects, the critical load calculated becomes occasionally very small. To overcome this problem, a slight damping matrix proportional to the stiffness

* Corresponding author. Tel.: +98 21 66164617; fax: +98 21 66022720.
E-mail addresses: mtorki@neo.tamu.edu, mtorki85@gmail.com (M.E. Torki), kazemi@sharif.edu (M.T. Kazemi), haddadpour@sharif.edu, haddadpour@gmail.com (H. Haddadpour), s.mahmoudkhani@yahoo.com (S. Mahmoudkhani).
[1] Tel.: +98 21 66164237; fax: +98 21 66014828.

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matrix can be used in the solution. Bochkarev and Matveenko [15] studied the dynamic stability of cylindrical shells conveying fluid for free–free and clamped-free boundary conditions by using the perturbation of velocity potential method. Biswas et al. [16] used a finite-element approach, based on the first-order shear theory (FST) and Sander's approximation, to study the effect of flaws on free vibration, buckling, and dynamic instability — both flutter and divergence — of cross-ply and angle-ply composite curved shell panels subjected to non-uniform, centrally-distributed, and edge-distributed follower forces. They evaluated the effects due to load type, load width, damage, and damage location on the natural frequency, buckling, divergence or flutter load, and flutter-threshold frequency. Their results demonstrated that that narrow-edge loading is undesirable in most cases.

Flutter of *FGMs* has been studied by many researchers. Considering the effect of temperature, Haddadpour et al. [17] predicted the flutter of *FGM* cylindrical shells under supersonic air flow by using the Classic Plate Theory (*CST*) and considering the von Karman nonlinearity. They observed that flutter load can become very small, even zero at temperatures near a specific critical temperature.

Although a large amount of work concerning the flutter of homogeneous cylindrical shells under follower forces is reported in the literature, to the best of the authors' knowledge, it seems that work on the corresponding problem for *FGM* shells is meager. Thus, the dynamic stability of *FGM* cylindrical shells under follower forces will be discussed in the sequel. For this purpose, Love's hypotheses along with *FST* are used to derive the differential equations of motion, and the extended Galerkin method is used to solve the equation systems. The problem is solved for two types of *FGM*, one hardening, and the other softening with power.

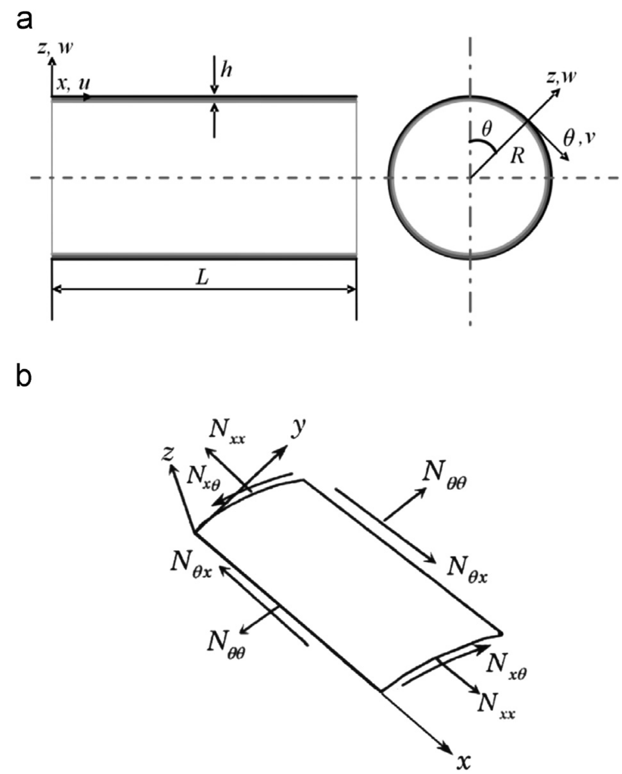

Fig. 1. (a) The coordinate system and (b) strain resultants considered for cylindrical shells [17,18].

2. Theoretical formulation

2.1. Functionally graded materials

For the *FGM* cylindrical shell, depending on whether the outer surface is pure metal or pure ceramic, the effective mechanical properties including elasticity modulus and Poisson's ratio, and physical parameters such as density, thermal expansion coefficient, and thermal conductivity can be obtained using either Eqs. (1) or (2) [18].

$$F_{eff}(z) = F_m V_m(z) + F_c[1 - V_m(z)] = (F_m - F_c)V_m(z) + F_c,\ V_m = \left(\frac{z}{h} + \frac{1}{2}\right)^N, N \geq 0 \quad (1)$$

$$F_{eff}(z) = F_c V_c(z) + F_m[1 - V_c(z)] = (F_c - F_m)V_c(z) + F_m,\ V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^N, N \geq 0 \quad (2)$$

where F_{eff} is the effective mechanical or physical property and F_m and F_c are the corresponding parameters for the metal and ceramic phases, respectively. Also, V_m and V_c stand for the volume fraction of metal and ceramic, respectively. The ceramic phase has greater elasticity modulus and lower density and Poisson's ratio compared to the metal phase [18]. Therefore, in the case that the volume fraction is defined by Eq. (1), the effective elasticity modulus of *FGM* increases with N; thus, it is called *hardening FGM* (by the author). All the same, if the volume fraction is defined using Eq. (2), the effective elasticity modulus is decreased with N, i.e. it is named *softening FGM*. Fig. 1 depicts the definition of hardening and softening *FGM*.

The properties of *FGM* materials are temperature-dependent. In the present research, the temperature has been assumed to be constantly equal to the reference temperature (the environment temperature), i.e. 300 K. In this case, the elasticity moduli, Poisson's ratios, and densities of nickel, stainless steel, and alumina will be obtained as included in Table 1 [19,20].

2.2. Constitutive equations

Consider a cylindrical shell with radius R, thickness h, and length L. In case that the coordinate system is taken to be as shown in Fig. 2a, then according to *FST*, the deformation components of any point can be written as [18]:

$$\begin{aligned} u(x,\theta,z,t) &= u_0(x,\theta,t) + z\phi_x(x,\theta,t) \\ v(x,\theta,z,t) &= v_0(x,\theta,t) + z\phi_\theta(x,\theta,t) \\ w(x,\theta,z,t) &= w_0(x,\theta,t) \end{aligned} \quad (3)$$

where u_0, v_0, and w_0 are the displacement components of the middle surface and ϕ_x and ϕ_θ are changes in the slope of the normal to the middle surface around θ and x axes, respectively. The stress resultants per unit length for a cylindrical shell are shown in Fig. 2b.

For the strain components, Love's hypotheses are used, which express the following [18]:

- The transverse normal is inextensible.
- Normals to the reference surface of the shell before deformation remain *straight*, but not necessarily *normal*, after deformation.
- Deflections and strains are infinitesimal.
- The transverse normal stress is negligible (plane-stress state is invoked).