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Large rotation theory for static analysis of composite and piezoelectric laminated thin-walled structures

S.Q. Zhang [a,b,*], R. Schmidt [a]

[a] Institute of General Mechanics, RWTH Aachen University, Templergraben 64, D-52062 Aachen, Germany
[b] School of Mechanical Engineering, Northwestern Polytechnical University, 710072 Xi'an, PR China

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ABSTRACT

A fully geometrically nonlinear finite element (FE) model is developed using large rotation shell theory for static analysis of composite and piezoelectric laminated thin-walled structures. The proposed large rotation theory is based on the first-order shear deformation (FOSD) hypothesis. It has six independent kinematic parameters which are expressed by five mechanical nodal degrees of freedom (DOFs). Linear electro-mechanically coupled constitutive equations with a constant electric field distribution through the thickness of each smart material layer are considered. Eight-node quadrilateral plate/shell elements with five mechanical DOFs per node and one electrical DOF per smart material layer are employed in the FE modeling. The present large rotation FE model is implemented into static analysis of both composite and piezoelectric laminated plates and shells. The equilibrium equation is solved by Newton–Raphson algorithm with system matrices updated in every iteration. The results are compared with those presented in the literature and others calculated by various simplified nonlinear shell theories. They indicate that large rotation theory has to be considered for the calculation of displacements and sensor output voltages of smart structures undergoing large deflections, since other simplified nonlinear theories fail to predict the static response precisely in many cases.

1. Introduction

Smart structures are those integrated with smart materials, *e.g.* piezoelectrics, electrostrictives, magnetostrictives, etc., acting as sensors and actuators in a feedback architecture. Due to a number of beneficial properties of thin-walled smart structures, the applications are greatly increasing in many fields of technology, like automotive and aerospace engineering, for vibration control, shape control, noise control, damage detection, health monitoring, among others.

The simulation of the static behavior of smart structures is essential for designing and manufacturing of smart structures. This requires models which are able to predict the static behavior of smart structures precisely. In contrast to three-dimensional (3-D) FE methods, see [1–7] among many others, which give very precise FE models but with large size of system matrices, one-dimensional (1-D) and two-dimensional (2-D) FE methods based on various hypotheses are much more frequently used, due to small model size and relative high accuracy.

The majority of papers in the literature proposed geometrically linear 1-D or 2-D FE models for static or dynamic analysis of electro-mechanically coupled problems based on various hypotheses, *e.g.* Bernoulli beam theory [8,9], Timoshenko beam theory [10], Kirchhoff-Love plate/shell theory which yields the so-called classical plate/shell theory [11–17], or Reissner–Mindlin plate/shell theory known as FOSD theory, see [18–25] among many others. In order to describe the transverse shear strain distribution in thickness direction more precisely, a third-order shear deformation (TOSD) or higher-order shear deformation (HOSD) hypothesis was first proposed by Reddy [26,27]. Later the theory was extended and applied to FE analysis of smart structures by Correia et al. [28], Correia et al. [29] and Moita et al. [30]. Furthermore, Loja et al. [31] and Soares et al. [32] presented higher-order B-spline FE strip models for laminated composite structures bonded with piezoelectric patches. More advanced shear deformation hypotheses, *e.g.* first-order zigzag [25,33], and third-order zigzag [34,35] shear deformation theories have been applied to smart structures as well.

Since linear models are only valid for problems at small strains and small rotations, geometric nonlinearity was taken into account on various levels for modeling of large deflections and large amplitude vibrations of thin-walled structures with isotropic, orthotropic or anisotropic materials. Reddy developed von Kármán type geometrically nonlinear FE models based on FOSD [36] and

* Corresponding author at: Institute of General Mechanics, RWTH Aachen University, Templergraben 64, D-52062 Aachen, Germany. Tel.: +49 241 8098286; fax: +49 241 8092231.
E-mail addresses: shunqi.zhang@hotmail.com, shunqi@iam.rwth-aachen.de (S.Q. Zhang).

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TOSD [37,38] hypotheses for composite laminated structures. A moderate rotation theory was first proposed by Librescu and Schmidt [39], Schmidt and Reddy [40], and later applied by Palmerio et al. [41,42] and Kreja et al. [43]. Considering more strain–displacement relations, Bouhafs et al. [44] and Klosowski and Woznica [45] developed fully geometrically nonlinear FE models of composite structures, which are, however, restricted to the range of moderate rotations. A number of papers proposed large or finite rotation theories for static analysis of composite structures, which can be found in [46–50] among others.

However, there are only a few papers which implemented nonlinear theories in modeling of thin-walled structures integrated with smart materials *e.g.* piezoelectric, electrostrictive or magnetostrictive materials. The papers that appeared recently mainly developed von Kármán type nonlinear FE models using different kinematic hypotheses for smart structures. Kapuria and Dumir [51] presented a von Kármán type nonlinear FE model based on classical plate theory. FE models using von Kármán type nonlinearity have been developed also based on FOSD [52,53] and TOSD [54,55] hypotheses. Additionally, a zigzag theory has been applied by Ray and Shivakumar [56] and Sarangi and Ray [57] with a layerwise FOSD hypothesis, and by Icardi and Sciuva [58] with a layerwise TOSD hypothesis, while Kapuria and Alam [59] proposed a first-order zigzag theory with a global third-order displacement variation. Applying moderate rotation theory, Lentzen et al. [60,61] developed nonlinear FE models for piezoelectric integrated smart structures based on FOSD hypothesis, which include more nonlinearities than von Kármán type nonlinear theory. Furthermore, fully geometrically nonlinear FE models have been developed by Moita et al. [62] based on classical theory, by Gao and Shen [63] and Kundu et al. [64] based on FOSD hypothesis, and by Dash and Singh [65] based on TOSD hypothesis. However, the fully geometrically nonlinear FE models presented in [62–65] are not real large rotation models, even though full geometric nonlinearities are included. In order to extend the analysis to smart structures undergoing large deflections and rotations, Chroscielewski et al. [66,67] developed one-dimensional (1-D) large rotation FE models for shape and vibration control of curved beams. Recently, Zhang and Schmidt [68] developed 2-D large rotation FE models for dynamic analysis of piezoelectric integrated smart plates and shells. In contrast to 1-D and 2-D FE methods, Marinković et al. [69,70] developed a degenerated shell element for fully geometrically nonlinear analysis of thin-walled piezoelectric structures. Yi et al. [71], Klinkel and Wagner [72,73] developed 3-D full nonlinear FE models for static and dynamic analyses of smart structures.

Concerning the nonlinear analyses of thin-walled smart structures, most of the studies are focusing on von Kármán type geometrically nonlinear theory and moderate rotation theory. A few papers took into account fully geometrically nonlinear theory, however, they use only five parameters (three mid-surface displacements and two mid-surface rotations) which is permitted only for small or moderate rotations, see [50,68]. Therefore, these nonlinear shell theories cannot predict static or dynamic behavior of smart structures precisely when the structures undergo large rotations. Based on the earlier work of Kreja and Schmidt [50], and Kreja [74] on large rotation nonlinear FE static analysis of isotropic or orthotropic laminated structures, the aim of this paper is to develop a large rotation nonlinear FE model for static analysis of piezoelectric laminated thin-walled smart structures based on FOSD hypothesis. The present implemented large rotation theory has six independent kinematic parameters, which are expressed by five mechanical nodal DOFs using Euler angle relations. A nonlinear static FE model, including an equilibrium equation and a sensor equation, is derived by the FE method and the principle of virtual work. Eight-node quadrilateral shell elements with five mechanical DOFs per node and one electrical DOF per smart layer are adopted in the FE analysis. In order to deal with locking effects, two element types are considered: SH851FI for full integration and SH851URI for uniformly reduced integration. The present FE model is tested by using a benchmark problem of a composite plate, and later applied to the simulation of piezoelectric coupled smart beams, plates and shells.

2. Strain field

The large rotation shell theory has six independent kinematic parameters expressed by five nodal DOFs, abbreviated as LRT56 theory, which can be found in [50,68]. Full geometrically nonlinear strain–displacement relations are considered in LRT56 theory. In order to clearly describe LRT56 theory, some basic vectors are introduced as shown in Fig. 1. Here, the Cartesian coordinate system (X^1, X^2, X^3) is fixed as global coordinates, and the curvilinear coordinates $(\Theta^1, \Theta^2, \Theta^3)$ are used to represent the geometry of the structures, which can be *e.g.* plate, cylindrical, spherical or any other coordinate systems. The position vectors of an arbitrary point in the shell space and at the mid-surface are denoted by $\mathbf{R}(\Theta^1, \Theta^2, \Theta^3)$ and $\mathbf{r}(\Theta^1, \Theta^2)$, respectively. The base vectors $\mathbf{g}_i$ are tangent vectors in the shell space, and $\mathbf{a}_\alpha, \mathbf{a}_3(\mathbf{n})$ are those at the mid-surface. The vectors in the deformed configuration are indicated by an overbar. In the undeformed configuration, $\mathbf{n}$ is a normal unit vector, and Θ^3 is a straight line. Latin indices represent the numbers 1, 2 or 3, and the Greek ones vary between 1 and 2. The relation between $\mathbf{R}$ and $\mathbf{r}$ in the undeformed configuration can be expressed as

$$\mathbf{R}(\Theta^1, \Theta^2, \Theta^3) = \mathbf{r}(\Theta^1, \Theta^2) + \Theta^3 \mathbf{n}. \tag{1}$$

Using the FOSD hypothesis, straight lines in thickness direction remain straight but not normal to the mid-surface after deformation. The relation between $\bar{\mathbf{R}}$ and $\bar{\mathbf{r}}$ in the deformed configuration reads as follows:

$$\bar{\mathbf{R}}(\Theta^1, \Theta^2, \Theta^3) = \bar{\mathbf{r}}(\Theta^1, \Theta^2) + \Theta^3 \bar{\mathbf{a}}_3. \tag{2}$$

According to the geometric relations displayed in Fig. 1, the displacement vector $\mathbf{u}$ can be obtained as

$$\mathbf{u} = \overset{0}{\mathbf{u}} + \Theta^3 \overset{1}{\mathbf{u}}, \tag{3}$$

with

$$\overset{0}{\mathbf{u}} = \bar{\mathbf{r}} - \mathbf{r} = \overset{0}{v}_\alpha \mathbf{a}^\alpha + \overset{0}{v}_3 \mathbf{n}, \tag{4}$$

$$\overset{1}{\mathbf{u}} = \bar{\mathbf{a}}_3 - \mathbf{n} = \overset{1}{v}_\alpha \mathbf{a}^\alpha + \overset{1}{v}_3 \mathbf{n}, \tag{5}$$

where $\overset{0}{v}_1, \overset{0}{v}_2, \overset{0}{v}_3$ are the translational displacements at the mid-surface referred to the contravariant base vectors $\mathbf{a}^\alpha$, $\mathbf{n}$, and $\overset{1}{v}_1, \overset{1}{v}_2, \overset{1}{v}_3$ are the generalized rotational displacements, *i.e.* the projections of $\overset{1}{\mathbf{u}}$ in the contravariant base vector triad of the undeformed mid-surface. Due to

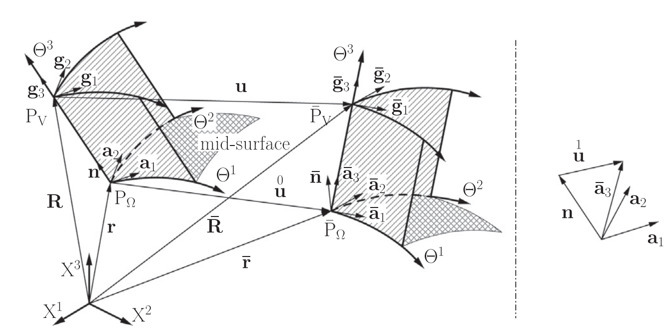

Fig. 1. Definition of base vectors.