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Nonlinear forced vibration of functionally graded cylindrical thin shells

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ABSTRACT

The nonlinear forced vibration of infinitely long functionally graded cylindrical shells is studied using the Lagrangian theory and multiple scale method. The equivalent properties of functionally graded materials are described as a power-law distribution in the thickness direction. The energy approach is applied to derive the reduced low-dimensional nonlinear ordinary differential equations of motion. Using the multiple scale method, a special case is investigated when there is a 1:2 internal resonance between two modes and the excitation frequency is close to the higher natural frequency. The amplitude–frequency curves and the bifurcation behavior of the system are analyzed using numerical continuation method, and the path leading the system to chaos is revealed. The evolution of symmetry is depicted by both the perturbation method and the numerical Poincaré maps. The effect of power-law exponent on the amplitude response of the system is also discussed.

1. Introduction

Functionally graded materials (FGMs) are composite materials made of two or more constituent phases with smoothly variable volume fraction. One of the remarkable advantages of FGMs is the elimination of stress discontinuity that is often encountered in laminated composites, and accordingly, delamination-related problems being avoided [1]. Many researchers and engineers believe that FGMs will play a very important role in many engineering applications. Numerous reports on the studies of functionally grade (FG) structures have been published [2]. As for vibration and dynamic problems of FG structures, many pioneering studies can be obtained from Loy et al. [3], Cheng and Batra [4], Reddy and Cheng [5], Yang and Shen [6–8]. Different types of higher order shear deformation theory are developed to investigate the gradient effects of FGMs in a precise manner. Yang and Shen [8], Huang and Shen [9] employed a higher order shear deformation shell and plate theory respectively to study the free vibration and dynamic response of FG cylindrical panels and plates. Zenkour [10] studied the buckling and free vibrations of FG sandwich plates based on sinusoidal shear deformation plate theory, and it was found that the classic plate theory showed a good accuracy for FG thin plates. Those else who consider the shear deformation in the study of vibration of FG structures include Matsunaga [11–13], Hosseini-Hashemi et al. [14], etc. Multi-layered method was used by Shakeri et al. [15] to describe the material gradient of FGMs approximately; and they analyzed the dynamic response of FG thick hollow cylinders using Galerkin finite element and Newmark methods. Meanwhile, some researchers investigate the dynamic problems of FG structures directly based on three-dimensional (3D) elastic theory. Malekzadeh and co-workers [16,17] analyzed 3D free vibration of FG thick annular plates and truncated conical shells. Vel [18] gave an exact elastic solution for the vibration of FG anisotropic cylindrical shells based on the 3D linear elastodynamics. Asgari and Akhlaghi [19] presented a natural frequency analysis of thick hollow cylinders made of two-dimensional (2D) FGM according to 3D equations of elasticity.

Pradyumna and Bandyopadhyay [20] investigated free vibration of FG curved panels using a higher-order finite element formulation. Oyekoya et al. [21] developed a Mindlin-type element and a Reissner-type element to study the buckling and vibration frequencies of FG rectangular plates. Talha and Singh [22] investigated the large amplitude free vibration frequencies of FG rectangular plates based on a nonlinear finite element formulation taking the higher order shear deformation into account. Tornabene and co-workers [23–25] analyzed the frequency characteristics of FG plates, annular plates, parabolic panels, conical and cylindrical shells, respectively, using generalized differential quadrature method based on a first order shear deformation theory; using the same method, Viola and Tornabene [26] discussed the natural frequencies of FG parabolic panels of revolution; in order to describe gradient properties of FGMs along thickness direction in a more precise manner, they developed a multi-parametric generalized power-law distribution function to model material properties of FGMs.

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Recently, Alijani and co-workers [27] studied the nonlinear forced vibrations of FG doubly-curved shallow shells with rectangular planform based on the Donnell's nonlinear shallow shell theory; bifurcation diagrams and the Poincaré maps were obtained, and chaotic regions were illustrated by calculating Lyapunov exponents and Lyapunov dimension. In another research [28], the authors discussed the impact of steady temperature distribution on the nonlinear vibrations of FG doubly curved shallow shells. Meanwhile, they [29] also investigated the nonlinear vibrations of FG plates in thermal environment, and revealed significant effect of non-linearity on the vibration of FG plates. Hao et al. [30] analyzed nonlinear dynamic behaviors of cantilever FG rectangular plates in thermal environment; the 1:1 internal resonance between first two modes and 1:2 subharmonic resonance were discussed using asymptotic perturbation method; and the numerical results showed that cantilever FG rectangular plates occurred periodic, quasi-periodic and chaotic motion in some given conditions.

This work deals with the nonlinear forced vibration of infinitely long FG cylindrical shells using the Lagrangian theory and multiple scale method. The properties of FGM are assumed to be graded in the thickness direction according to a simple power-law distribution. Donnell's nonlinear shell theory and energy approach are employed to derive the reduced low-dimensional nonlinear ordinary differential equations of motion of FG cylindrical shells. The complicated response and bifurcation characteristics are discussed for a 1:2 internal resonance between two modes and a higher-frequency primary resonance excitation occurring simultaneously. The phenomenon of so-called symmetry breaking and restoring, and chaotic motion are predicted by both perturbation method and numerical Poincaré maps.

2. Basic equations

2.1. Energy formulation

A FG cylindrical thin shell with mid-surface radius R and thickness h is considered in a reference frame of cylindrical coordinate system, where x is longitudinal, θ circumferential, z normal (positive inwards); and w is the deformation along radial direction, as shown in Fig. 1.

Assume that the FG cylindrical shell is made of two constituent materials. And the effective properties P (Young's modulus E, Poisson's ratio ν, mass density ρ) are considered to be graded in thickness direction according to a power-law distribution:

$$P(z) = (P_1 - P_2)\left(\frac{h-2z}{2h}\right)^N + P_2 \tag{1}$$

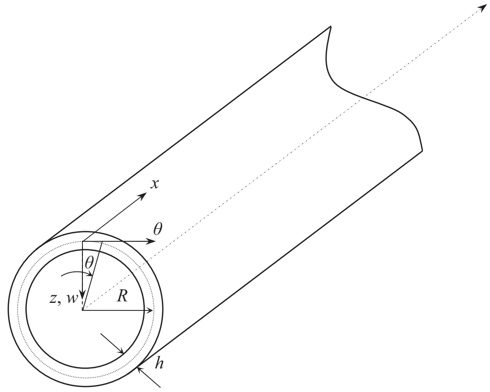

Fig. 1. Geometry of FG cylindrical shell and reference coordinate.

where P_1 and P_2 are respectively the properties of constituent material, the subscripts 1 and 2 indicate constituent 1 and constituent 2 respectively; the superscript N is the power-law exponent, $N \in [0, \infty)$, reflecting the volume fraction of constituent 1. According to Eq. (1), the inner surface ($z = h/2$) is constituent 2 rich whereas the outer surface ($z = -h/2$) is constituent 1 rich. Here, we introduce some material moduli which will be used in the analysis:

$$\bar{\rho} = \int_{-h/2}^{h/2} \rho(z)\mathrm{d}z \tag{2a}$$

$$(D_0^*, D_1^*, D_2^*) = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu(z)^2}(1, z, z^2)\mathrm{d}z,$$
$$(\overline{D}_0^*, \overline{D}_1^*, \overline{D}_2^*) = \int_{-h/2}^{h/2} \frac{\nu(z)E(z)}{1-\nu(z)^2}(1, z, z^2)\mathrm{d}z \tag{2b}$$

Based on Donnell's nonlinear shell theory, the strain-displacement relations defined in the cylindrical coordinate frame can be written as [31]

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x, \varepsilon_\theta = \varepsilon_\theta^0 + z\kappa_\theta, \gamma_{x\theta} = \gamma_{x\theta}^0 + z\kappa_{x\theta} \tag{3}$$

where ε_x and ε_θ are strain components along x and θ direction, respectively, $\gamma_{x\theta}$ is shear strain in $x\theta$ plane. ε_x^0, $\varepsilon_\theta^0$and $\gamma_{x\theta}^0$ are the membrane strains, defined as:$\varepsilon_x^0 = u_{,x} + w_{,xx}/2$, $\varepsilon_\theta^0 = (v_{,\theta} - w)/R + (w_{,\theta})^2/2R^2$, $\gamma_{x\theta}^0 = v_{,x} + u_{,\theta}/R + w_{,x}w_{,\theta}/R$; while κ_x, κ_θ and $\kappa_{x\theta}$ are the curvatures, given by: $\kappa_x = -w_{,xx}$, $\kappa_\theta = -w_{,\theta\theta}/R^2$ and $\kappa_{x\theta} = -2w_{,x\theta}/R$, where a comma denotes differentiation with respect to x or/and θ variables. Meanwhile, according to Donnell's nonlinear shell theory the membrane force resultants (N_x, N_θ, $N_{x\theta}$) of a cylindrical shell can be written as

$$N_x = D_0^*\varepsilon_x^0 + \overline{D}_0^*\varepsilon_\theta^0 + D_1^*\kappa_x + \overline{D}_1^*\kappa_\theta$$
$$N_\theta = \overline{D}_0^*\varepsilon_x^0 + D_0^*\varepsilon_\theta^0 + \overline{D}_1^*\kappa_x + D_1^*\kappa_\theta$$
$$2N_{x\theta} = (D_0^* - \overline{D}_0^*)\gamma_{x\theta}^0 + (D_1^* - \overline{D}_1^*)\kappa_{x\theta} \tag{4}$$

where the moduli are defined respectively as

$$D_0 = \frac{D_0^*}{(D_0^*)^2 - (\overline{D}_0^*)^2}, \quad \overline{D}_0 = -\frac{\overline{D}_0^*}{(D_0^*)^2 - (\overline{D}_0^*)^2} \tag{5a}$$

$$D_1 = \frac{D_0^*D_1^* - \overline{D}_0^*\overline{D}_1^*}{(D_0^*)^2 - (\overline{D}_0^*)^2}, \quad \overline{D}_1 = \frac{D_0^*\overline{D}_1^* - \overline{D}_0^*D_1^*}{(D_0^*)^2 - (\overline{D}_0^*)^2} \tag{5b}$$

$$D_2 = D_2^* - \frac{D_0^*[(D_1^*)^2 + (\overline{D}_1^*)^2] - 2\overline{D}_0^*D_1^*\overline{D}_1^*}{(D_0^*)^2 - (\overline{D}_0^*)^2},$$
$$\overline{D}_2 = \overline{D}_2^* - \frac{2D_0^*D_1^*\overline{D}_1^* - \overline{D}_0^*[(D_1^*)^2 + (\overline{D}_1^*)^2]}{(D_0^*)^2 - (\overline{D}_0^*)^2} \tag{5c}$$

Reversing Eq. (4), the membrane strains can be expressed as functions of membrane force resultants and curvatures

$$\varepsilon_x^0 = D_0N_x + \overline{D}_0N_\theta - D_1\kappa_x - \overline{D}_1\kappa_\theta$$
$$\varepsilon_\theta^0 = \overline{D}_0N_x + D_0N_\theta - \overline{D}_1\kappa_x - D_1\kappa_\theta$$
$$\gamma_{x\theta}^0 = 2(D_0 - \overline{D}_0)N_{x\theta} - (D_1 - \overline{D}_1)\kappa_{x\theta} \tag{6}$$

Introducing the in-plane Airy stress function φ, such that $N_x = \varphi_{,\theta\theta}/R^2$, $N_\theta = \varphi_{,xx}$ and $N_{x\theta} = -\varphi_{,x\theta}/R$. Then the compatibility equation is obtained as

$$D_0\nabla^2\nabla^2\varphi = \nabla_1^2 w - \overline{D}_1\nabla^2\nabla^2 w - \frac{1}{2}\ell(w, w) \tag{7}$$

here, $\nabla^2(\bullet) = (\bullet)_{,xx} + (\bullet)_{,\theta\theta}/R^2$ and $\nabla_1^2(\bullet) = (\bullet)_{,xx}/R$, the nonlinear operator $\ell(\alpha, \beta)$ is defined as $\ell(\alpha, \beta) = (\alpha_{,xx}\beta_{,\theta\theta} + \beta_{,xx}\alpha_{,\theta\theta} - 2\alpha_{,x\theta}\beta_{,x\theta})/R^2$.

Instead of obtaining the governing equation, energy expressions for both kinetic and potential energy are gained to characterize a thin