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Nonlinear free vibration of skew nanoplates with surface and small scale effects

P. Malekzadeh *, M.R. Golbahar Haghighi, M. Shojaee

Department of Mechanical Engineering, School of Engineering, Persian Gulf University, Bushehr 7516913798, Iran

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ABSTRACT

The nonlinear free flexural vibration of skew nanoplates is studied by considering the influences of free surface energy and size effect (small scale) simultaneously. The formulations are derived based on classical plate theory (CPT) in conjunction with nonlocal and surface elasticity theories using Hamilton's principle. Green's strain tensor together with von Kármán assumptions is employed to model the geometrical nonlinearity. The free surfaces are modeled as two-dimensional membranes adhering to the underlying bulk material without slipping. The solution algorithm is based on the transformation of the governing differential equation from the physical domain to a rectangular computational one, and discretization of the spatial derivatives by employing the differential quadrature method (DQM) as an efficient and accurate numerical tool. The effect of small scale parameter and surface effect together with the geometrical parameters and boundary conditions on the nonlinear frequency parameters of the skew nanoplates are studied.

1. Introduction

The nanostructural elements such as nanowires, nanorods, nanobeams and nanoplates have been used as the building blocks in modern science and technology in recent years, for example, in micro/nano electromechanical systems (MEMS/NEMS) [1] and biosensors [2]. This is because they have superior mechanical, electrical and thermal performances with respect to the conventional structural elements. On the other hand, since the thin nanoplates can undergo large deformation without violating the failure criteria, the nonlinear analysis becomes essential to accurately achieve the dynamic behavior of these nanostructural elements for engineering design and manufacture.

Both experimental observations [3–5] and atomistic simulations [6–8] indicate that when the structural size decreases towards the nanoscale regime, their mechanical properties and consequently their response become size-dependent. This is because the surface layers differ from their bulk counterparts in that their elastic responses are intrinsically size-dependent and the surface-to-bulk energy ratio increases with the increase of surface-to-volume ratio. Hence, the surface energy becomes a significant part of the total elastic potential energy and should be taken into account when studying the mechanical behavior of this type of elements, which is usually neglected in the classical elasticity theory. On the other hand, the size (small scale) effect, which is due to long-range interatomic interaction, has significant influence on the mechanical behavior of the nanostructural elements and should be considered to achieve solutions with acceptable accuracy [8].

Both surface and small scale effects inherently appear in the governing equations when one uses atomistic simulation methods to analyze the nanostructural elements. But, these methods consume much time and are computationally expensive for analyzing nanostructures with large numbers of atoms. Thus, because of the simplicity, computational efficiency and accuracy, continuum mechanic approaches are often adopted; see for example Refs. [9–29]. To overcome the drawbacks of the classical continuum theory in predicting the size dependence (free surface energy) and small scale effect of material properties of nanostructures, usually the surface elasticity theory of Gurtin and Murdoch [30,31] and the nonlocal elasticity theory of Eringen [32–34] are employed, respectively.

Based on the surface elasticity theory of Gurtin and Murdoch [30,31], the surfaces are modeled as two-dimensional membrane adhering to the underlying bulk material without slipping to account the effect of surfaces/interfaces on mechanical properties. It has been shown that with correct choice of surface elastic properties, this surface elasticity theory explains various size-dependent phenomena at the nanoscale and the predictions fit well with atomistic simulations and experimental measurements [4,35,36]. Also, the nonlocal elasticity theory of Eringen is implemented by assuming that the stress at a point is a function of the

* Corresponding author. Tel.: +98 771 422 2150; fax: +98 771 454 0376.
E-mail addresses: malekzadeh@pgu.ac.ir, p_malekz@yahoo.com (P. Malekzadeh).

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strains at all points in the domain and as a result, the inter-atomic forces and atomic length scales directly appear in the constitutive relations as material parameters [32–34].

In recent years, some studies have been performed to investigate the surface effects on the bending and the linear and nonlinear free vibration of nanoplates; see for example Refs. [21–25]. However, in these interesting studies the nonlocal effect has not been taken into account. On the other hand, based on the nonlocal constitutive relation of Eringen and without including the surface effects, some researchers have been attempted to develop nonlocal nanoplate models and apply them to analyze linear and nonlinear vibration behaviors of nanoplates; see for example Refs. [9–20]. To the best of authors′ knowledge, the work of Wang and Wang [29] is the only one which includes both the surface and nonlocal effects on the linear free vibration behavior of nanoplates. They employed both the classical as well as the first-order shear deformation theories to study the linear free vibration behavior of simply supported rectangular nanoplates.

From the literature survey, it can be seen that the surface and nonlocal effects on the nonlinear free vibration analysis of rectangular nanoplates have been investigated separately. On the other hand, the previous studies showed that both these phenomenon have significant effects on the vibrational behavior of nanoplates. Hence, in this paper, the nonlinear free vibration analysis of skew nanoplates is performed by considering both the surface as well as the small scale effects simultaneously. The formulation is derived based on the classical plate theory (CPT) in conjunction with the von Kármán geometrical nonlinearity assumptions. Since it is not possible to obtain the analytical solution for the nonlinear free vibration analysis of skew nanoplates with arbitrary boundary conditions, the approximate methods should be employed to solve the problem. In addition, the DQM is an efficient and accurate numerical approach compared to the weighted residual methods [37–43]. Hence, this numerical method is employed to solve the nonlinear differential governing equation of skew nanoplates subjected to arbitrary boundary conditions. The effects of surface elasticity, nonlocal parameter, skew angle and amplitude ratio of the skew nanoplates on the nonlinear natural frequency of the skew nanoplates are studied.

2. Mathematical modeling and solution procedure

Consider a skew nanoplate of length a, width b and thickness h as shown in Fig. 1. It is assumed that the nanoplate is homogeneous and isotropic. A Cartesian coordinate system (x,y,z) is used to label the material points of the skew nanoplate in the undeformed reference configuration. Since the large deformation effect is practically important for thin nanoplates, the CPT seems to be sufficient for modeling of the dynamic behavior of the thin nanoplates. Based on the CPT, the displacement components $\overline{u}$ (in the x-direction), $\overline{v}$ (in the y-direction) and $\overline{w}$ (in the z-direction) of an arbitrary material points of the nanoplate can be approximated as,

$$\overline{u}(x,y,z,t) = u(x,y,t) - z\frac{\partial w}{\partial x}, \tag{1a}$$

$$\overline{v}(x,y,z,t) = v(x,y,t) - z\frac{\partial w}{\partial y}, \tag{1b}$$

$$\overline{w}(x,y,z,t) = w(x,y,t) \tag{1c}$$

where u, v and w are the in-plane displacement components along the x and y-directions and the transverse displacement component of the material point (x,y) on the mid-surface of the nanoplate. Using Eq. (1) and based on the von Kármán assumptions, the nonzero components of Green′s strain tensor in terms of the displacement components become,

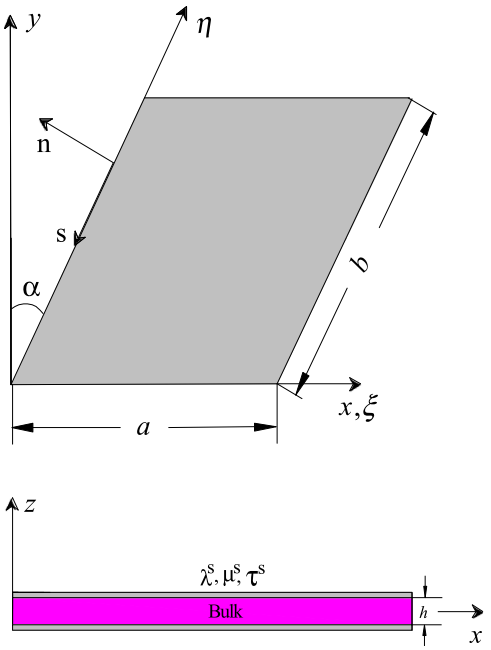

Fig. 1. The geometry of skew nanoplate.

$$\varepsilon_{xx} = \varepsilon^0_{xx} - z\frac{\partial^2 w}{\partial x^2}, \tag{2a}$$

$$\varepsilon_{yy} = \varepsilon^0_{yy} - z\frac{\partial^2 w}{\partial y^2}, \tag{2b}$$

$$\gamma_{xy} = \gamma^0_{xy} - 2z\frac{\partial^2 w}{\partial x \partial y}, \tag{2c}$$

where $\varepsilon_{ii}(i=x,y)$ and γ_{xy} are the normal and shear strain tensor components of an arbitrary material point of the nanoplate, respectively; also, $\varepsilon^0_{ii}(i=x,y)$ and γ^0_{xy} are the normal and shear strain tensor components of an arbitrary material point on the mid-plane of the nanoplate, respectively,

$$\varepsilon^0_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \tag{3a}$$

$$\varepsilon^0_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \tag{3b}$$

$$\gamma^0_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}. \tag{3c}$$

The constitutive relations of the upper (S^+) and lower (S^-) surface layers as given by Gurtin and Murdoch [30,31] can be expressed as

$$\sigma^{\pm}_{xx} = \tau^{\pm}_0 + (2\mu^{\pm}_0 + \lambda^{\pm}_0)\varepsilon^{\pm}_{xx} + (\lambda^{\pm}_0 + \tau^{\pm}_0)\varepsilon^{\pm}_{yy}, \tag{4a}$$

$$\sigma^{\pm}_{yy} = \tau^{\pm}_0 + (2\mu^{\pm}_0 + \lambda^{\pm}_0)\varepsilon^{\pm}_{yy} + (\lambda^{\pm}_0 + \tau^{\pm}_0)\varepsilon^{\pm}_{xx}, \tag{4b}$$

$$\sigma^{\pm}_{xy} = (2\mu^{\pm}_0 - \tau^{\pm}_0)\gamma^{\pm}_{xy} \tag{4c}$$

Here, $\tau^{\pm}_0$ are the residual surface tension under unstrained condition, $\mu^{\pm}_0$ and $\lambda^{\pm}_0$ the surface Lame′s constants, $\varepsilon^{\pm}_{ii}$ and $\gamma^{\pm}_{ij}$ $(i,j=x,y)$ the normal and shear strain components of the surfaces $S^{\pm}$, respectively. Without loss of generality, it is assumed that the surfaces $S^{\pm}$ have the same material properties,

$$\lambda^{\pm}_0 = \lambda^s, \tag{5a}$$