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Buckling and vibration of shear deformable functionally graded orthotropic cylindrical shells under external pressures

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ABSTRACT

In this study, the vibration and buckling of functionally graded (FG) orthotropic cylindrical shells under external pressures is investigated using the shear deformation shell theory (SDST). The basic equations of shear deformable FG orthotropic cylindrical shells are derived using Donnell shell theory and solved using the Galerkin method. Parametric studies are made to investigate effects of shear deformation, orthotropy, compositional profiles and shell characteristics on the dimensionless frequency parameter and critical external pressures. Some comparisons among various theories have been performed in order to show the differences between the parabolic shear deformation theory (PSDT) and several higher-order shear deformation theories (HSDTs).

1. Introduction

The shells of anisotropic and composite materials require simple and reliable models that would reflect an increased sensitivity of such materials to the transverse shear deformation. An adequate class of such shell theories, known as shear deformation theories, has been developed, from the ideas originated by Reissner [1] and Mindlin [2] in the context of plates. After these studies, various shear-deformation theories were developed to improve the analysis of the vibration and buckling of plates and shells, and this has led to more accurate results. Ambartsumian [3] proposed a transverse shear stress function in order to explain plate deformation. Dong and Tso [4] developed a constitutive relation for laminated orthotropic shells which includes a transverse shear deformation. Reddy and Liu [5] developed a simple higher order shear deformation shell theory (HOST), in which the transverse shear strains are assumed to be parabolically distributed across the shell thickness and which contains the same dependent unknowns as in the first order shear deformation theory (FSDT). Later, some new functions were proposed Soldatos and Timarci [6], Karama et al. [7] and Aydogdu [8].

The shear deformation plays a significant role in the buckling and vibration behaviors of shells composed of traditional and new generation composites. As the effect of shear deformation is not considered, it can lead to 30% or more errors for the frequencies and buckling loads of homogeneous composite cylindrical shells. Thus, shear deformable shell theory (SDST) becomes more interesting than classical shell theory (CST). In the open literature, there are some important publications on the solution of vibration of homogenous orthotropic cylindrical shells in which different types of shear deformable shell theories are used. Soedel [9] investigated the vibration of shells with Timoshenko-Mindlin type shear deflections and rotary inertia. Bhimaraddi [10] proposed a higher-order theory for free vibration analysis circular cylindrical shells. Soldatos [11] developed the thickness shear deformation theories for the dynamic analysis of non-circular cylindrical shells. Tong [12] investigated effects of transverse shear deformation on free vibration of orthotropic conical shells. Timarci and Soldatos [13] proposed a unified shear deformable shell theory (USDST) for dynamic studies of symmetric cross-ply circular cylindrical shells. Al-Khatib and Buchanan [14] investigated free vibration of a paraboloidal shell of revolution including shear deformation and rotary inertia. Ferreira et al. [15] presented the vibration analysis of laminated shells by the sinusoidal shear deformation theory (SSDT) and radial basis functions collocation, accounting for through-the-thickness deformations. Pradyumna and Bandyopadhyay [16] presented dynamic instability behavior of laminated hypar and conoid shells using a higher-order shear deformation theory. Civalek [17] studied vibration analysis of laminated composite conical shells by the method of discrete singular convolution based on the shear deformation theory. Viola et al. [18] examined general higher-order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels.

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In the past few decades, numerous publications on the buckling analysis of homogeneous anisotropic materials have appeared. However, investigations involving the application of shear-deformable shell theories for the buckling analysis are limited in number. Shirakawa [19] investigated effects of shear deformation and rotary inertia on the buckling and vibration of cylindrical shells. Palazotto and Linnemann [20] studied the buckling and vibration characteristics of composite cylindrical panels incorporating effects of a higher-order shear theory. Kardomateas [21] presented Koiter-based solution for the initial postbuckling behavior of moderately thick orthotropic and shear deformable cylindrical shells under external pressure. Eslami and Shariyat [22] developed a higher order shear deformation theory to study the dynamic buckling and postbuckling of thick composite cylindrical shells and the solution was sought on the basis of numerical methods. Shen [23] used the boundary layer theory for the buckling and post-buckling of an anisotropic laminated cylindrical shell with the shear deformation under the external pressure. Li and Lin [24] studied the buckling and post-buckling of shear deformable anisotropic composite cylindrical shell subjected to various external pressure loads. Ferreira et al. [25] investigated buckling analysis of isotropic and laminated plates by radial basis functions according to a higher-order shear deformation. Asadi and Qatu [26] presented static analysis of thick laminated shells with different boundary conditions, using two first order shear deformation theories (FSDTs). Ádány [27] examined flexural buckling of simply-supported thin-walled columns with consideration of membrane shear deformations, based on shell model.

Previous studies have shown that the shear deformation plays a significant role in the buckling and vibration behavior of homogeneous composite shells. As the combined effects of inhomogeneity of materials, shear deformation and rotary inertia are taken into account in the dynamic equations of composite cylindrical shells, it is extremely difficult to obtain closed form solutions for stability and vibration problems of FG shells with anisotropic properties. Functionally graded materials (FGMs) are inhomogeneous with spatially varying material properties. Owing to its special features with potential applications to many science and engineering fields, the FGM structure has attracted wide and increasing attentions of scientists and engineers in broad areas of research. So far, a number of reviews dealing with various aspects of FGMs have been published [28,29]. Due to the increased relevance of the FGM cylindrical shells in the design of aerospace structures, their buckling and vibration characteristics with account taken of combined effects of compositional profiles of FG materials and shear deformation have attracted the attention of many scientists [30–36]. The functionally graded isotropic materials are considered in the previous studies. Few works have been done for the stress, buckling and vibration analyses of FG structural elements made of anisotropic materials [37–47]. All these studies are based on the classical shell theory (CST) in which use the Kirchhoff–Love hypothesis, i.e., the transverse shear deformation is neglected.

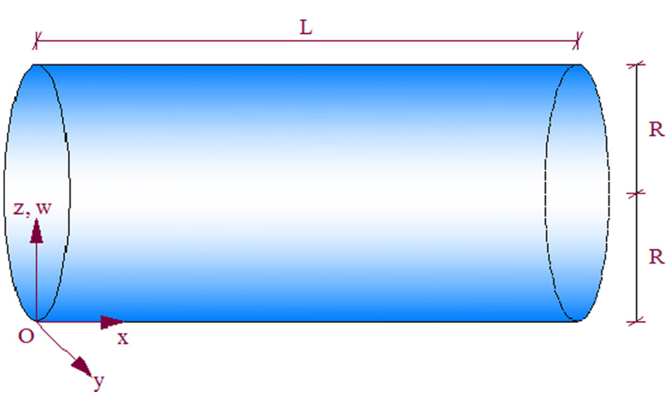

Fig. 1. Nomenclature and coordinate system of a circular cylindrical shell.

Therefore, it is very important to develop an accurate, reliable analysis towards the understanding of the buckling and vibration characteristics of shear deformable functionally graded orthotropic shell structures. In this work, the vibration and buckling analyses of FG orthotropic cylindrical shells subjected to external pressures are investigated using SDST and CST. The basic equations of FG orthotropic cylindrical shells with account taken of shear deformation and rotary inertia are derived using the Donnell shell theory and solved using the Galerkin method. Parametric studies are made to investigate effects of transverse shear deformation and FG compositional profiles of the materials with the variation of shell characteristics on the frequency parameter, and critical hydrostatic and lateral pressures. The results are verified by comparing the obtained values with those in the existing literature. In additional, some comparisons among various theories have been performed in order to show the differences between the parabolic shear deformation theory (PSDT) and several higher-order shear deformation theories (HSDTs).

2. Formulation of the problem

Consider a circular cylindrical shell with mean radius R, length L and thickness h, which is made from the functionally graded orthotropic material. The cylindrical shell geometry and the curvilinear coordinate system employed are shown in Fig. 1. The shell is referred to a coordinate system ($Oxyz$) in which x and y are in the axial and circumferential directions of the cylindrical shell and z is in the direction of the inward normal to the reference surface. The origin of the coordinate system is located at the end of the shell on the reference surface. The displacement is w and the angles of rotation of a normal to the reference surface are φ and ψ, respectively.

The material gradient (inhomogeneity) of orthotropic cylindrical shell is assumed to arise due to the exponential variation of Young's moduli, shear moduli and density along the thickness direction. Hence, the Young's moduli, shear moduli and density can be expressed as exponential functions of Z, the normalized coordinate in the thickness direction, as follows [37]:

$$E_1(Z)=E_{01}e^{\mu(Z-0.5)},\quad E_2(Z)=E_{02}e^{\mu(Z-0.5)},\quad G_{12}(Z)=G_{012}e^{\mu(Z-0.5)},$$
$$G_{13}(Z)=G_{013}e^{\mu(Z-0.5)},\quad G_{23}(Z)=G_{023}e^{\mu(Z\pm0.5)},\quad \rho(Z)=\rho_0e^{\mu(Z-0.5)},$$
$$Z=z/h \tag{1}$$

where E_{01} and E_{02} are Young's moduli of the homogeneous orthotropic material along the x and y directions, respectively; $G_{012}, G_{013}, G_{023}$ are shear moduli which characterize angular chances between principal directions x and y, x and z, y and z, respectively; ρ_0 is the density of the homogeneous orthotropic material and μ is the exponential factor characterizing the degree of material gradient in z direction and satisfying $-1\le\mu\le1$. We remark that $\mu=0$ corresponds to the homogeneous case, $\mu>0$ to the graded soft material, and $\mu<0$ to the graded stiff material.

3. Governing relations

The stress–strain relationships for FG orthotropic cylindrical shells with the shear deformation my be expressed by [37,48]

$$\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix}=\begin{bmatrix}B_{11}(Z) & B_{12}(Z) & 0\\ B_{21}(Z) & B_{22}(Z) & 0\\ 0 & 0 & B_{66}(Z)\end{bmatrix}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{pmatrix} \tag{2.1}$$