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Geometric imperfections and lower-bound methods used to calculate knock-down factors for axially compressed composite cylindrical shells

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ABSTRACT

The important role of geometric imperfections on the decrease of the buckling load for thin-walled cylinders had been recognized already by the first authors investigating the theoretical approaches on this topic. However, there are currently no closed-form solutions to take imperfections into account already during the early design phases, forcing the analysts to use lower-bound methods to calculate the required knock-down factors (KDF). Lower-bound methods such as the empirical NASA SP-8007 guideline are commonly used in the aerospace and space industries, while the approaches based on the Reduced Stiffness Method (RSM) have been used mostly in the civil engineering field. Since 1970s a considerable number of experimental and numerical investigations have been conducted to develop new stochastic and deterministic methods for calculating less conservative KDFs. Among the deterministic approaches, the single perturbation load approach (SPLA), proposed by Hühne, will be further investigated for axially compressed fiber composite cylindrical shells and compared with four other methods commonly used to create geometric imperfections: linear buckling mode-shaped, geometric dimples, axisymmetric imperfections and measured geometric imperfections from test articles. The finite element method using static analysis with artificial damping is used to simulate the displacement controlled compression tests up to the post-buckled range of loading. The implementation of each method is explained in details and the different KDFs obtained are compared. The study is part of the European Union (EU) project DESICOS, whose aim is to combine stochastic and deterministic approaches to develop less conservative guidelines for the design of imperfection sensitive structures.

1. Introduction

Since the beginning of 1900s researchers developing formulations for the buckling of thin-walled cylinders, e.g. Southwell (1914) [1], have observed a discrepancy between theoretical and experimental results. In particular, the measured buckling loads were typically much lower than the corresponding predicted buckling loads of a geometrically perfect cylinder. Southwell found that his theory could not be applied for real cases where there are geometric imperfections and load asymmetries. Flügge (1932) [2] and Donnell (1934) [3] were the first authors to develop formulations taking into account the effects of initial geometric imperfections, but the non-linear analyses failed to predict the experimental buckling loads. Their analyses required the use of large-magnitude geometric imperfections, that "could scarcely have escaped the notice of the investigators" [4]. Flügge's and Donnell's theories produce a gradual appearance of buckles with increasing the compression load, whereas in the experiments, buckling is typically characterized by a sudden dynamic buckling event and corresponding reduction in load. Koiter's theory (1945, which was translated from Dutch to English in the 1960s by Riks) was the first to predict accurately the imperfection sensitivity trends that were observed experimentally [4]. In 1950 Donnell and Wan [5], independently from the study of Koiter, modified the procedure adopted by Donnell [3] sixteen years earlier and proposed a new method, which was followed by several investigators with some modifications [6]. Arbocz (1992) [7] states that the Koiter's "General Theory of Elastic Stability" is widely accepted. However, it is important to mention that Koiter's theory is valid if the elasticity limit is not exceeded anywhere in the material [8]. In addition, the theory is based on an asymptotic expansion about the bifurcation point and is limited to small-magnitude imperfections and the range of validity is generally unknown.

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In the meanwhile, the design of imperfection sensitive structures required guidelines explaining how to take imperfection sensitivity into account, for instance, in the calculation of rocket and launcher structures [9]. In 1960, Seide, Weingarten and Morgan (see [10,11]) published a collection of experimental results which was one of the main precursors for the well-known NASA SP-8007 guideline, published in 1965 and revised in 1968 to its most popular version [12]. Fig. 1 shows this collection of experimental results and the lower-bound curve that gives the shell buckling knock-down factor denoted by γ in Eq. (1.1). Calculating γ for isotropic unstiffened cylinders requires only the cylinder radius and the wall thickness, as shown in Eq. (1.1). This equation also shows the equivalent thickness t_{eq} that is often used when calculating the KDF for orthotropic materials. However, note that approach for calculating knockdown factors for orthotropic materials does not consider all the orthotropic stiffness terms such as membrane-bending coupling, the two laminate directions and tension-shear are not included. These stiffness terms can have a significant influence on the buckling behavior, consequently on the resulting knock-down factors, as demonstrated by Geier et al. (2002) [13].

$$\gamma = 1 - 0.902(1 - e^{-\phi})$$
$$\phi = \frac{1}{16}\sqrt{\frac{R}{t}}\ \text{(isotropic)}$$
$$\phi = \frac{1}{16}\sqrt{\frac{R}{t_{eq}}}\quad \text{with} \quad t_{eq} = 3.4689\sqrt[4]{\frac{\overline{D}_{11}\overline{D}_{22}}{\overline{A}_{11}\overline{A}_{22}}}\quad \text{(orthotropic)} \tag{1.1}$$

$\overline{A}_{11}$, $\overline{A}_{22}$, $\overline{D}_{11}$ and $\overline{D}_{22}$ are the extensional and bending stiffnesses extracted from the composite ABD matrix.

In the NASA SP-8007 guideline the KDF denoted by γ in Eq. (1.1) is called correlation factor, accounting for the disparity between experiments and theory. Theoretical equations for the buckling load for both isotropic and orthotropic cylinders are also provided in NASA SP-8007, where the correlation factor is used, but in modern applications of γ, the theoretical buckling load is usually calculated using linear buckling analysis and the design load obtained multiplying this theoretical buckling load by γ as shown in Eq. (1.2).

$$F_{design} = F_{theoretical} \cdot \gamma \tag{1.2}$$

The Reduced Stiffness Method (RSM) developed by Croll (1972) [14], Batista & Croll (1979) [15] and collaborators is another method for calculating lower-bounds. Croll & Batista (1981) [16] used this concept to find lower-bounds for axially compressed linear-elastic isotropic cylinders. It has been mostly applied in the civil engineering field [17] and it is based on three postulates summarized as follows [18]: (1) significant geometric nonlinearities appear due to changes in membrane resistance. For instance, in the buckling of an in-plane loaded plate there is no non-linearity up to the point where some disturbance causes a normal deformation. The normal deformation causes a load eccentricity that creates bending, interacting non-linearly with the reduction of the membrane stiffness, which decreases in the post-buckled range of loading. In any case where a thin-walled structural member is initially under high compressive stress levels, with a high membrane component of the strain energy, the displacements are predicted linearly up to the point where the membrane stiffness starts to decrease. (2) For thin-walled structures, the post-buckling loss of stiffness can only occur when there is membrane resistance at the pre-buckled state, meaning that if the shell does not have membrane energy prior to buckling there will be no loss of stiffness after buckling. (3) The lower-bound buckling load for a particular load case will be given by an analysis in which the membrane stiffness is removed.

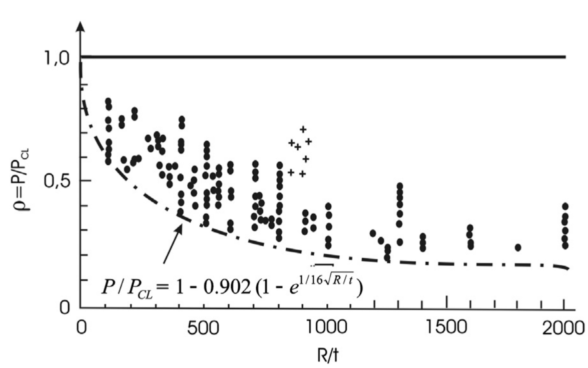

Fig. 1. Test data for isotropic cylinders subjected to axial compression (modified from Arbocz and Starnes Jr. [9]).

Sosa et al. (2006) [19] showed the equivalence between the reduced stiffness method and the reduced energy method (REM). Along this study the REM will be implemented in a general finite element solver following the procedure explained by Sosa et al., in which a reduction factor α is applied to the membrane stiffness components instead of completely eliminating it, as originally proposed by Croll [18]. This approach assumes that the shell with degraded membrane stiffness will have a post-buckled shape similar to an eigenvector obtained through linear buckling analysis. Sosa and Godoy (2010) [20] compared the REM using this assumption with non-linear post-buckling analysis and showed that this assumption may not be valid in some cases, leading to non-conservative estimatives. In such cases the computation of correction coefficients is required, making the REM less straightforward. The KDF using the REM is defined in Eq. (3.4). The implementation of the REM is explained in detail in Section 3.6.

Comparative studies performed by Hilburger et al. (2004) [21], Hühne et al. (2005) [22] and (2008) [23] and Degenhardt et al. (2008) [24] have shown that the lower-bound given by the NASA SP-8007 guideline can lead to conservative designs. Moreover, the space industry experience has shown that structures designed for buckling using the NASA SP-8007 guideline can be so conservative that, when tested after manufactured, fail for strength. Hühne suggested the single perturbation load approach (SPLA) as a robust method for creating a single buckle imperfection, referred herein as SPLI. Using the classification given by Winterstetter and Schmidt (2002) [25], Hühne classifies the SPLI as a "worst", "realistic" and "stimulating" imperfection. Another way of producing such single buckles is by directly translating the nodes in the finite element mesh. Wullschleger (2002) [26], suggests a simple model for these geometric dimple imperfections (GDI). Section 3.3 gives more details about this formulation and its implementation. Fig. 2 shows a typical knock-down curve obtained with the SPLA, where it can be seen that the buckling load (P) becomes nearly constant after a given level of single perturbation load (SPL), called minimum perturbation load (P1).

The idea of a single buckle as a worst imperfection was firstly pointed out by Esslinger (1970) [27] using high-speed cameras. Deml and Wunderlich (1997) [28] came to the same conclusion using a modified finite element formulation in which the nodal positions are treated as extra degrees-of-freedom (DOFs) which will vary along the solution within a pre-set amplitude. In this optimization problem for which the lowest buckling load is sought

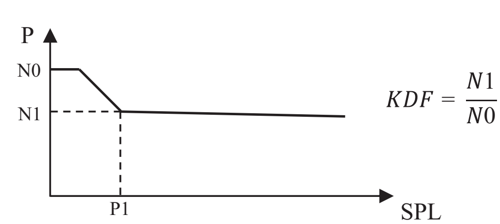

Fig. 2. Typical Knock-Down curve obtained with the SPLA.