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Nonlinear free vibration analysis of a plate-cavity system

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ABSTRACT

Free vibration of a plate-cavity system is analytically studied in this paper. For this purpose, a rectangular enclosure composed of one flexible and five rigid walls are taken into account. The flexible wall is modeled by the Von-Karman plate theory and the Galerkin method is employed to derive interior acoustic pressure and subsequent equations of motion. Harmonic balance approach, variational iteration method (VIM) and direct integration method are employed to determine nonlinear natural frequencies of the coupled system. A parametric study is then carried out and effects of different parameters on the value of frequency ratio are studied.

1. Introduction

Vibration of a plate backed by an air cavity has been one of the interesting research areas in recent years. One could address varieties of applications including vehicular and railway coach cabins, aircraft fuselages/skin panels, acoustical instruments and different aerospace structures. Different aspects of dynamic response of a cube enclosure have been already addressed in the literature. Pretlove [1] studied free vibration of a rectangular panel backed by a closed cavity using an analytical method. Linear natural frequencies of the coupled system were obtained by a matrix iteration technique in that study. Free vibration of a rectangular plate-cavity system was investigated by Qaisi [2]. Natural frequencies and mode shapes of the system were determined and effect of the cavity depth on the fundamental frequency was examined for different boundary conditions. Nakanishi et al. [3] derived a closed form expression for plate-cavity system by employing Helmholtz integral. They investigated the effect of different air cavity parameters on the plate vibration and sound field inside the coupled system. Kim and Kim [4] studied the interaction between a flexible structure and cavities of infinite and semi-infinite sizes. Structural-acoustic coupling influence on the natural frequency of an enclosure with one flexible wall was studied by Lee et al. [5]. They developed a theoretical model of a semi-cylindrical enclosure with a flexible panel [6]. Sound absorption of a flexible panel in a plate-cavity system has been also targeted in recent years. Lee et al. [7] developed an absorption criterion based on the modal analysis approach and compared the theoretical and experimental results. Jump phenomenon was captured in a panel-cavity system and effect of nonlinearity on the sound absorption properties was studied by Lee et al. [8]. Other different methods have been also used to investigate the problem of plate-cavity system. Green's function method, mixed finite element and classical formulation, impedance-mobility approach and probabilistic approach were employed by different researchers [9–12]. Interaction between a circular plate and a cylindrical fluid cavity was studied by Gorman et al. and Hasheminezhad et al. [13,14]. Mohamady et al. [15] examined the interior noise of an enclosure transmitted by a flexible panel. They used a harmonic point source outside the system and then predicted the eigenfrequencies of the coupled system by use of the finite element method. Resonance frequencies of a plate-cavity system have been analyzed by other procedures such as mode summation and harmonic balance method [16,17]. Active control of an enclosure composed of one flexible and five rigid walls has been also addressed in the literature [18–21]. More recently, sound absorption of a curved panel backed by a cavity was studied by Lee et al. [22].

Coupled nonlinear equations of motion for the plate modes can appropriately describe the problem of interaction between the air cavity and a flexible wall. Among the investigations provided in the literature, vibrating modes of the plate structure are assumed to be uncoupled with the others. In the present study, nonlinear differential equations of a plate backed by an air cavity are studied using an analytical method. It is shown that coupling between modes of the system has a significant effect on the results. It is shown that in addition to the cavity-plate coupling, nonlinear coupling between the mode shapes should be taken into account.

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Variational iteration method (VIM) is employed to solve the nonlinear equations of the system. Nonlinear natural frequencies are also determined by use of the harmonic balance approach. Results are compared in time and frequency domains. Sensitivity of the frequency–amplitude relationship with respect to different parameters is then examined in a parametric study.

2. Mathematical modeling

2.1. Problem formulation

Interaction between a flexible plate and air cavity is mathematically modeled in this section. Schematic configuration of the system is shown in Fig. 1. The model is composed of a flexible plate of length a, width b and thickness h, five rigid walls and an acoustic enclosure of depth c. It is assumed that the flexible panel vibration is governed by the Von-Karman plate theory. Partial differential equations of the plate can be presented in terms of its displacement and the Airy stress function as follows [23]:

$$D\nabla^4 W(x,y,t)+\rho\frac{\partial^2 W(x,y,t)}{\partial t^2} = P_E(x,y,t)-P_i(x,y,t)+h\left(\frac{\partial^2\phi}{\partial y^2}\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2\phi}{\partial x^2}\frac{\partial^2 W}{\partial y^2}-2\frac{\partial^2\phi}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\right) \tag{1}$$

$$\nabla^4\phi = E\left[\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2-\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right] \tag{2}$$

in which $W(x,y,t)$ and ϕ represent the plate displacement and the Airy stress function, respectively.ρ is the plate density, D and E denote bending stiffness and Young's modulus of the plate and $P_E(x,y,t)$, and $P_i(x,y,t)$ are the external excitation and acoustic pressure of the air cavity at $z=-c$. In this study, it is assumed that the plate is simply supported at its boundaries (SSSS) and the consequent boundary conditions can be presented by [24]

$$x=0,\ a \Rightarrow \begin{cases} W=0, & M_x=0 \\ \frac{\partial^2\phi}{\partial x\partial y}=0, & \int_0^b \frac{\partial^2\phi}{\partial y^2}dy=0 \end{cases} \tag{3}$$

$$y=0,\ b \Rightarrow \begin{cases} W=0, & M_y=0 \\ \frac{\partial^2\phi}{\partial x\partial y}=0, & \int_0^a \frac{\partial^2\phi}{\partial x^2}dx=0 \end{cases} \tag{4}$$

Using eigenfunction expansion (Galerkin method), one can assume the plate displacement and stress function to be

$$W=(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(t)\sin\frac{n\pi x}{a}\ \sin\frac{m\pi y}{b} \tag{5}$$

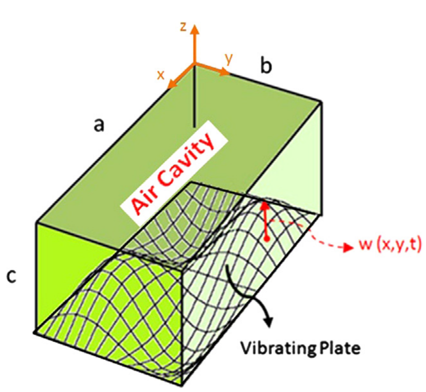

Fig. 1. Schematic configuration plate-cavity system.

$$\phi(x,y,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\phi_{mn}(t)\cos\frac{n\pi x}{a}\ \cos\frac{m\pi y}{b} \tag{6}$$

Substituting Eqs. (5) and (6) into Eqs. (1) and (2) results in the following equations:

$$\begin{aligned}
&\sum_m\sum_n((\rho\ddot{W}_{mn}(t)+D(\alpha_n{}^2+\beta_m{}^2)^2W_{mn}(t))\times(\sin\ \alpha_n x\times\ \sin\ \beta_m y))\\
&\quad=P_E(x,y,t)-P_i(x,y,t)\\
&\quad+h\Bigg[\left(\sum_m\sum_n\phi_{mn}(t)\times\beta_m{}^2\ \cos\ \alpha_n x\times\ \cos\ \beta_m y\right)\\
&\quad\times\left(\sum_m\sum_n W_{mn}(t)\times\alpha_n{}^2\ \sin\ \alpha_n x\times\ \sin\ \beta_m y\right)\\
&\quad+\left(\sum_m\sum_n\phi_{mn}(t)\times\alpha_n{}^2\ \cos\ \alpha_n x\times\ \cos\ \beta_m y\right)\\
&\quad\times\left(\sum_m\sum_n W_{mn}(t)\times\beta_m{}^2\ \sin\ \alpha_n x\times\ \sin\ \beta_m y\right)\\
&\quad-2\left(\sum_m\sum_n\phi_{mn}(t)\times\alpha_n\beta_m\times\ \sin\ \alpha_n x\times\ \sin\ \beta_m y\right)\\
&\quad\times\left(\sum_m\sum_n W_{mn}(t)\times\alpha_n\beta_m\times\ \cos\ \alpha_n x\times\ \cos\ \beta_m y\right)\Bigg]
\end{aligned} \tag{7}$$

$$\begin{aligned}
&\sum_m\sum_n(\alpha_n{}^2+\beta_m{}^2)^2\times\phi_{mn}(t)\times\ \cos\ \alpha_n x\times\ \cos\ \beta_m y\\
&\quad=E\times\Bigg[\left(\sum_m\sum_n W_{mn}(t)\times\alpha_n\beta_m\times\ \cos\ \alpha_n x\times\ \cos\ \beta_m y\right)^2\\
&\quad-\left(\sum_m\sum_n W_{mn}(t)\times\alpha_n{}^2\ \sin\ \alpha_n x\times\ \sin\ \beta_m y\right)\\
&\quad\times\left(\sum_m\sum_n W_{mn}(t)\times\beta_m{}^2\ \sin\ \alpha_n x\times\ \sin\ \beta_m y\right)\Bigg]
\end{aligned} \tag{8}$$

where $\alpha_n=n\pi/a$, and $\beta_m=m\pi/b$. Three modes of vibration i.e. (1,1), (1,2) and (2,1) are taken into account. Expanding Eqs. (7) and (8), and multiplying the equations by $\sin\ n\pi x/a\ \sin\ m\pi y/b$ and $\cos\ n\pi x/a\ \cos\ m\pi y/b$ and then by integrating it over the plate area one can arrive at the general equation of motion

$$\begin{aligned}
&M_{mn}\ddot{W}_{mn}(t)+K_{mn}W_{mn}(t)\\
&\quad=\int_0^b\int_0^a P_E(x,y,t)\times\ \sin\ \frac{n\pi x}{a}\ \sin\ \frac{m\pi y}{b}dxdy\\
&\quad-\int_0^b\int_0^a P_i(x,y,t)\times\ \sin\ \frac{n\pi x}{a}\ \sin\ \frac{m\pi y}{b}dxdy+(nonlinear\ terms)
\end{aligned} \tag{9}$$

In order to obtain the plate displacement from the latter equation, one needs to determine the interior acoustic pressure at $z=-c$, $P_i(x,y,t)$. Acoustic pressure within the air cavity is governed by the wave equation:

$$\begin{aligned}
&\nabla^2P(x,y,z,t)-\frac{1}{c_a{}^2}\frac{\partial^2P(x,y,z,t)}{\partial t^2}=0\\
&\Rightarrow\frac{\partial^2P}{\partial x^2}+\frac{\partial^2P}{\partial y^2}+\frac{\partial^2P}{\partial z^2}-\frac{1}{c_a{}^2}\frac{\partial^2P}{\partial t^2}=0
\end{aligned} \tag{10}$$

in which c_a is the sound speed inside the cavity. Using separation of variables technique, the internal pressure is given by

$$P(x,y,z,t)=X(x)Y(y)Z(z)T(t) \tag{11}$$

The corresponding boundary conditions for the internal pressure can be presented by

$$\left.\frac{\partial P}{\partial x}\right|_{x=0,\ a}=0,\quad \left.\frac{\partial P}{\partial y}\right|_{y=0,\ b}=0$$